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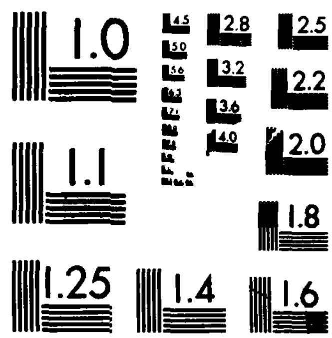

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A COMPARATIVE ANALYSIS OF A GENERALIZED
LANCHESTER EQUATION MODEL
AND
A STOCHASTIC COMPUTER SIMULATION MODEL

by

Terry A. West

March 1987

Thesis Advisor Don E. Harrison, Jr.

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AD A 183

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b RESTRICTIVE MARKINGS:

2a SECURITY CLASSIFICATION AUTHORITY:

2b DECLASSIFICATION / DOWNGRADING SCHEDULE:

3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited.

4 PERFORMING ORGANIZATION REPORT NUMBER(S):

5 MONITORING ORGANIZATION REPORT NUMBER(S):

6a NAME OF PERFORMING ORGANIZATION: Naval Postgraduate School

6b OFFICE SYMBOL (If applicable): 74

7a NAME OF MONITORING ORGANIZATION: Naval Postgraduate School

6c ADDRESS (City, State, and ZIP Code): Monterey, California 93943-5000

7b ADDRESS (City, State, and ZIP Code): Monterey, California 93943-5000

8a NAME OF FUNDING / SPONSORING ORGANIZATION:

8b OFFICE SYMBOL (If applicable):

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER:

8c ADDRESS (City, State, and ZIP Code):

10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification): A COMPARATIVE ANALYSIS OF A GENERALIZED LANCHESTER EQUATION MODEL AND A STOCHASTIC COMPUTER SIMULATION MODEL

12 PERSONAL AUTHOR(S): West, Terry A.

13a TYPE OF REPORT: Master's Thesis

13b TIME COVERED: FROM ______ TO ______

14 DATE OF REPORT (Year Month Day): 1987 March

15 PAGE COUNT: 145

16 SUPPLEMENTARY NOTATION:

17 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number): Lanchester Equation Theory, Combat Attrition Modeling, Deterministic Models, Stochastic Models, C^3 Modeling

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This thesis involved a numerical experiment to compare a deterministic Generalized Lanchester Equation model, referred to as the M/W model, and a stochastic computer simulation model, referred to as the C/S model. A discussion of the historical background of Lanchester's equations precedes the presentation of the two models and the experimental design. The results are presented graphically and show that the M/W force level trajectory is a good approximation for the C/S force level trajectory. It was also shown that the two model's trajectories behaved similarly. These results indicate that deterministic attrition models may often be good approximations for the mean of stochastic attrition models. Command and control applications of a model like the M/W model, are presented and a list of suggested follow-on research is provided to stimulate further work in this area.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21 ABSTRACT SECURITY CLASSIFICATION: Unclassified

22a NAME OF RESPONSIBLE INDIVIDUAL: Prof. Don E. Harrison, Jr.

22b TELEPHONE (Include Area Code): (408) 646-2877

22c OFFICE SYMBOL: 61Hx

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Approved for public release; distribution is unlimited.

A Comparative Analysis of a Generalized Lanchester Equation Model and a Stochastic Computer Simulation Model

by

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Captain, United States Army
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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN SYSTEMS TECHNOLOGY
(Command, Control and Communications)

from the

NAVAL POSTGRADUATE SCHOOL
March 1987

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ABSTRACT

This thesis involved a numerical experiment to compare a deterministic Generalized Lanchester Equation model, referred to as the M/W model, and a stochastic computer simulation model, referred to as the C/S model. A discussion of the historical background of Lanchester's equations precedes the presentation of the two models and the experimental design. The results are presented graphically and show that the M/W force level trajectory is a good approximation for the C/S force level trajectory. It was also shown that the two model's trajectories behaved similarly. These results indicate that deterministic attrition models may often be good approximations for the mean of stochastic attrition models. Command and control applications of a model like the M/W model, are presented and a list of suggested follow-on research is provided to stimulate further work in this area.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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ACKNOWLEDGEMENTS

I would like to express a special thank you to John M. Wozencraft, Distinguished Professor Emeritus, for his generous assistance. His knowledge and encouragement was extremly helpful in the completion of this thesis.

I. INTRODUCTION

A. PURPOSE AND OVERVIEW

The main purpose of this thesis is to conduct an experiment, using two different types of combat attrition models, to determine whether their results are similar. There is a belief that deterministic Lanchester Equation type models closely parallel the mean results of stochastic computer simulations. This research will attempt to determine whether the results of a deterministic "Generalized Lanchester Equation" type model and a stochastic computer simulation model are similar. This work is the first such research of this type published to the author's knowledge.

Chapter One provides the reader with an introduction to modeling and an overview of the history of Lanchester Equation type models. The purpose of Chapter Two is to introduce the two models which are utilized in this experiment and Chapter Three is a presentation of the experimental design. The results of the experiment are provided in Chapter Four and a discussion of the conclusions and recommended follow-up research are given in Chapter Five.

B. MODELING

Throughout history, military analysts have been developing and utilizing various types of models for detailed analysis to assist in decision making. A model is defined as a "simplified representation of the entity it imitates or simulates" [Ref. 1: p. 1]. A second definition of a model used by the U.S. Army Models Review Committee, "an abstract representation of reality which is used for the purpose of prediction and to develop understanding about the real-world processes" [Ref. 2: p. 5], implies that models are designed to be close representations of a real-world entity or process. A final definition for a **military model** is, "an abstraction of reality, the elements of which are chosen for (a) an investigative purpose or (b) a resource management purpose; in other words, an abstraction to assist in making decisions." [Ref. 1: p. 3]. From these definitions, it can be seen how important the development and use of models is to the military analyst.

In general, there are three types of models:

1) Iconic model
2) Analogue model
3) Symbolic model

An iconic model is a miniature version of the entity, such as an airplane or a tank model. An analogue model is an artificial representation of reality. An example of an analogue model is a map which represents the three dimensional real-world on a two dimensional, small scale sheet of paper. A symbolic model is one in which words or numerical descriptions are used to represent an entity or process. An example of a symbolic model is a mathematical equation which represents a process such as attrition.

The types of models used by military analysts cover the entire spectrum from military field exercises (which can be thought of as iconic models) to analytical models. Figure 1.1 is a combination of information found in Taylor's book *Force-on-Force Attrition Modelling* [Ref. 2: p. 7], and Hughes' book *Military Modeling* [Ref. 1: p. 10], which shows the various types of models in use and their characteristics of operational realism, degree of abstraction, convenience and accessibility.

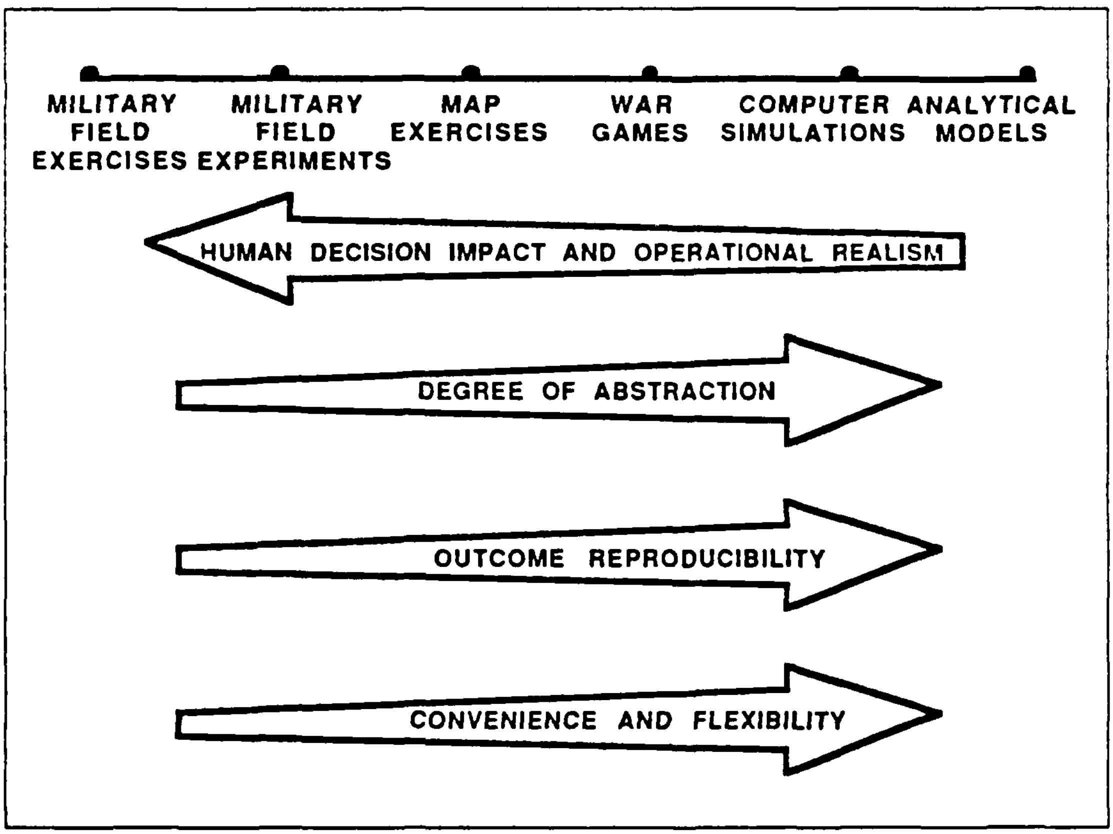

Figure 1.1 Current Model Types.

The three most convenient and accessible types of models shown in Figure 1.1 are war games, computer simulations and analytical models. These three types of models are also the most commonly used to model combat attrition. A brief description of each of them will be given to better understand and help distinguish between them.

1. **War Games**

War games are conducted using real-world combat scenarios and people playing various positions on a headquarters battle staff. A war game is conducted to allow a commander and his staff to exercise the staff decision making processes in 'realistic' combat scenarios. This allows the commander to exercise and develop his staff without incurring the true cost of combat, the loss of human life. War games can be run utilizing the sand table method or on computers. The key to war gaming is that there is always a person making the decisions that are required in the game.

2. **Computer Simulations**

Computer simulations are also used to model the combat attrition process. The primary difference between computer simulations and war games is the method used to represent the decision process. War games use people playing staff roles to make the decisions where simulations use algorithms to represent the decision process. A combat simulation not only represents the combat process, it acts it out from start to finish. A computer simulation begins with a set of input parameters and runs continuously until completion of the battle and then provides the results of the battle as output. Computer simulations which utilize pseudo-random number generators to determine the results of random events, such as the outcome of one soldier firing at another, are called Monte Carlo Simulations and are stochastic models.

3. **Analytical Models**

Analytical models are the third type of model used in modeling combat attrition. Analytical models are symbolic models which use mathematical symbols and equations to represent the combat attrition process. Analytical models can be developed as stochastic models or deterministic models. Stochastic models, as introduced in the previous section, utilize probablity distributions to determine certain variables. Therefore the output from two consecutive runs of a stochastic model will more than likely be different. The deterministic model however, will produce the same set of output values for a given set of input parameters.

My research is focusing on the comparison of a stochastic **Monte Carlo Simulation** and a deterministic analytical model. The deterministic model I am using for comparison is a **Generalized Lanchester Equation** model which was developed from the original Lanchester Equation Theory. To provide a basis for the readers understanding of this experiment, I will give a brief introduction to the history of the Lanchester Equation type models.

C. LANCHESTER EQUATION THEORY

1. History Of Lanchester Equation Type Modeling

F. W. Lanchester was a British aeronautical engineer, who in 1914 developed two sets of simple differential equations to model the combat attrition process between two opposing homogeneous forces (i.e., fighter aircraft vs.. fighter aircraft). His intentions were to provide insight into the dynamics of combat under 'modern conditions' of warfare and to justify the principle of concentration of forces. The term modern conditions of warfare will be addressed again later.

Lanchester's original work was designed to model a force-on-force attrition process involving two homogeneous forces. He developed two combat attrition models, each having it's own unique assumptions, for this purpose. These original models were designed to model an 'aimed fire' combat scenario and an 'area fire' combat scenario. These two models are often referred to as the classical Lanchester Equation Theory.

2. Modeling Aimed Fire

The phrase **aimed fire,** as Lanchester used it in his original work, refers to combat between two forces, X and Y, where each combatant from X force acquires a Y force target (i.e., locates and takes aim) and fires. An example of this type combat is an infantry battle for control of an area such as hill 224. The following assumptions apply for this model:

1) The combat being modeled involves two homogeneous forces (i.e., Infantry vs. Infantry).
2) The entire X force and Y force are within weapons range of one another.
3) The effects of weapons rounds are independent.
4) Each of the forces is well enough aware of the location and condition of all enemy forces so that they will engage only live enemy units. Also the rate at which they kill enemy targets is constant.
5) Fire is uniformly distributed over surviving enemy targets.

Lanchester's original model for **aimed fire** is given by Equation Set 1.1:

$$dx/dt = -ay(t) \qquad \text{with } x(0) = x_0,$$
$$dy/dt = -bx(t) \qquad \text{with } y(0) = y_0, \qquad \text{(eqn 1.1)}$$

where $x(t)$ is defined as the number of combatants of X force at time t, $y(t)$ is defined as the number of combatants of Y force at time t, and a and b are constants which are called attrition-rate coefficients. The attrition-rate coefficient 'a' represents the effectiveness of a Y combatant killing a X combatant per unit of time. Similarly the attrition-rate coefficient 'b' represents the effectiveness of a X combatant killing a Y combatant per unit of time. This set of equations assumes that the rate at which a force X can cause casualties to a Y force is proportional to the number of combatants in the X force. Similarly, the rate at which a combatant from Y force can cause casualties to the X force is assumed to be proportional to the number of combatants in the Y force. When Equation Set 1.1 is integrated, the resulting state equation is obtained and has been labeled the **Lanchester Square Law:**

$$b(x_0^2 - x(t)^2) = a(y_0^2 - y(t)^2). \qquad \text{(eqn 1.2)}$$

3. Modeling Area Fire

The phrase **area fire** as Lanchester used it in his original work, refers to a combat scenario involving two homogeneous forces, say X and Y, which are uncertain of exact enemy locations and engage one another by firing in the general area where the enemy force is located. This scenario has the following assumption set:

1) The combat being modeled involves two homogeneous forces (i.e., Infantry vs. Infantry)
2) The entire X force and Y force are within weapons range of one another.
3) The effects of weapons rounds are independent.
4) Each force is aware only of the general location of the enemy force and therefore engages the enemy by firing into that general 'area' without the benefit of knowing their effectiveness.
5) Fire from all surviving combatants is uniformly distributed over the area which the enemy occupies.
6) Each force has the same vulnerable area to enemy fire.

Lanchesters original model for **area fire** is defined by the Equation Set 1.3:

$$dx/dt = -axy,$$

$$dy/dt = -byx. \qquad \text{(eqn 1.3)}$$

This model assumes that the rate of attrition is proportional to both force levels $x(t)$ and $y(t)$. This means that in the **area fire** model, the rate at which a X force can cause casualties to a Y force is not only dependant upon the number of combatants in its own force, but also on the number of combatants alive in the Y force. The rate at which the Y force can cause casualties on the X force is similarly dependent on both their own force level and the force level of the opposing force X.

The well known **'Linear Law'** is obtained by integration of Equation Set 1.3 and is shown here in Equation 1.4 :

$$b(x_0-x(t)) = a(y_0-y(t)), \qquad \text{(eqn 1.4)}$$

where x_0 and y_0 are the initial force levels at time $t=0$, $x(t)$ and $y(t)$ are the force levels at time t, and a and b are the attrition-rate coefficients for each force.

4. **Attrition Rate Coefficients**

The Lanchester attrition-rate coefficients were introduced in the models above and were defined as constants. In order to use the Lanchester models there must exist a set of acceptable attrition-rate coefficients for each possible type of engagement. Lanchester's original model utilizing homogeneous forces, only requires the availability of a limited number of coefficients. However, in Lanchester Equation type models which allow for the combat between heterogeneous forces, the number of attrition-rate coefficients grows very rapidly when one considers all the possible combinations of combatant weapons vs. combatant weapons that can occur. As Lanchester postulated his original work, he used constants for attrition-rate coefficients, which implies that the kill capability of a force does not change over time. With constant attrition-rate coefficients these models are easy to solve. However, there is no reason why we should believe that such factors as range, weather, visibility, and training do not affect the kill capability of a combatant. Taylor provides two methods for determining numerical values for Lanchester attrition-rate coefficients which are used in the United States. [Ref. 2: p. 45]

1) A statistical estimate based on 'combat' data generated by a detailed Monte Carlo combat simulation.

2) An analytical submodel of the attrition process for the particular combination of firer and targets.

In the first method, the simulation output is used to fit one or more free parameters in the analytical model in an attempt to have it to provide similar results. S. Bonder has labeled this approach a 'fitted-parameter analytical model' [Refs. 2,3: pp. 45,73-88].

The second method for developing the attrition-rate coefficients has been labeled an 'independant analytical model' by S. Bonder [Refs. 2,3: pp. 47,73-88]. The concept for developing the coefficients is to consider a single firer shooting at a single target (which is stationary and does not fire back). These coefficients are developed under perfect conditions with this method. There has been a large amount of work done in the development of acceptable attrition-rate coefficients. Taylor provides a good list of references in his book *Force-on-Force Attrition Modeling* [Ref. 2].

5. **Modern Conditions of Warfare**

In 1914 when F. W. Lanchester developed his original models, he was attempting to model the combat attrition process so that he could learn more about the dynamics of combat under 'modern conditions'. The phrase modern conditions was an important one then as it is today. In earlier sections Lanchesters models were introduced and the assumptions that applied to them were given. The assumption sets are what tailor the model to the conditions. Some of these original assumptions no longer seem appropriate to model what we would term 'combat under modern conditions'. The next section discusses some of the shortcomings of the original Lanchester Models, but before that it is important to lay out a solid definition for combat under modern conditions.

Today the conditions on the combat battlefield are very complex. First, the air and land battles are thought of as one battle and given the title 'AIR-LAND BATTLE'. The ground forces commander from the battalion level on up, has a combination of different combat units under his direct command. An Army Division Commander has Infantry, Armor, and Artillery Battalions all assigned to his command. The battalions which are assigned are not 'pure' units, they too have a combination of combat units assigned. This concept has been adopted by the Army as the way to fight the next war. The Army has organized under the 'combined arms' concept and believe that placing a mix of supporting combat forces together on the battlefield will be a force multiplier.

The Navy has developed much the same type of conditions at sea. The Carrier Battle Groups (CBG) are combinations of many types of naval combat power and therefore brings a variety of forces together to fight a battle at one point under the Officer-in-Tactical-Command (OTC).

6. Shortcomings With The Original Lanchester Models

Lanchester's original models provided a very simplified model of modern warfare which has served as the basis for a great deal of further work in this area. There are many shortcomings of the original models which have been identified and have stimulated much of the work done in the years following their development. Lanchesters models have been popular because of their simplicity and the ease of tracing through the mathematical computations. In this section I will discuss some of the shortcoming which have been identified in applying Lanchesters original laws to model today's combat.

First, not all of today's combat scenarios can be neatly placed into the 'square' or 'linear' attrition models. The most often referenced and easily explained example is a guerilla ambush scenario. In this case it is assumed that the guerilla force is heavily camouflaged and the force being attacked is in the open. It is apparent that the attacked force would suffer a 'squared' attrition rate from the aimed fire of the guerilla force which has clear line of sight of the entire force under attack. On the other hand, the defending force would not have clear line of sight of the attacking guerilla force and therefore would not be properly modeled by a 'square' attrition model, but rather the 'linear' attrition model. However, even this does not properly model the scenario. With time the defending force will be able to locate the attacking force well enough that their fire should be considered as 'aimed fire' and modeled by a 'squared' attrition model. This illustrates the dynamic conditions on the battlefield which must be considered. [Ref. 4]

One of the main assumptions of Lanchester's laws is that the two opposing forces are comprised of homogeneous units (i.e., Infantry vs.. Infantry). This is rarely the case on the battlefield today. Today the doctrine for a land battle is the Air-Land Battle Doctrine, and is written for combined arms units. The combined arms concept was introduced earlier, but warrants reemphasis. The force that must be modeled today combines several different combat units under one commander to fight a battle.

An additional consideration is the close relationship between the ground force and the close air support provided by the Air Force which adds another unit type into the model. The same type of unit composition is currently found in the Navy Carrier Battle Groups. The need for a model which allows for several different types of units (the heterogeneous force model) is obvious. The use of heterogeneous forces requires the model to consider allocating fire power on a variety of targets. The factors that enter in the allocation process are; the attrition rate of each type of weapon for each of the opposing forces weapons types, the number and type of enemy targets, and the relative significance of each of the enemy targets.

Another assumption of the original Lanchester theory is that all of the forces of both sides are committed to battle at the beginning and there are no reinforcements available. Furthermore, there is no allowance for the possibility of withdrawing some or all of a unit's forces. These factors are definitly consider by the commander on the battlefield today.

Separation of forces on the battlefield is ignored in the original models. They assume that the units remain fixed in location and are always within range of the opposing forces weapons. To better model the combat process, a movement factor which would affect the weapons' ranges and the accuracy of each weapon on the battlefield would be required.

The original model is designed to model a 'fight to the finish' type of combat. This definition for battle termination, of fighting to the last man, is not realistic on today's battlefield. There is some point in a battle where the defender will decide to withdraw or surrender before the entire force is destroyed. [Ref. 4]

With this introduction to Lanchester Equation type models as a basis, the next chapter will present the two models used for this research.

II. THE MODELS

A. GENERAL

Two models were used for this research, one is a deterministic model and the other is a stochastic computer simulation model. The deterministic model used was developed at the Naval Postgraduate School by Professor Paul H. Moose and Professor John M. Wozencraft. Their model is a Generalized Lanchester Equation type model. It will be referenced throughout the remainder of this thesis as the M/W model or the Moose/Wozencraft model.

The stochastic computer simulation model which was utilized for this research is a Monte Carlo Simulation written by Professor Don E. Harrison, Jr. at the Naval Postgraduate School. It will be referenced throughout the remainder of the thesis as the C/S model or the computer simulation model.

In the following sections, each of these models will be presented to provide a general understanding of there characteristics.

B. MOOSE/WOZENCRAFT MODEL

Paul H. Moose is a professor in the Electrical Engineering Department at the Naval Postgraduate School and John M. Wozencraft was a professor with the Electrical Engineering Department prior to his retirement. Both have been associated with the academic group for the Joint Command, Control, and Communications Curriculum, and have been interested in the decision and control problems a modern day military commander faces. They wanted to develop a better understanding of how command and control decisions affect battle outcome. To do this, an analytical model of the attrition process that would provide an adequate representation of the modern conditions of combat was needed. Such a model required modeling a variety of forces as an aggregate force fighting on the battlefield. [Ref. 5]

As presented in chapter one, the military commander in today's environment has several types of units available to him to fight a battle. The question of optimum resource allocation involves the use of a variety of these units. For the model to be useful it had to be easy to interpret and understand. Lanchester Equation type models were easy to interpret, but were limited in their ability to model a variety of forces. This motivated their research into the dynamics of a Generalized System of Lanchester Equations of the type shown here in Equation Set 2.1:

$$\dot{x}_i(t) = F_i(x_i, y, t): \qquad \text{where } i = 1,2,\ldots\ldots,N,$$

(eqn 2.1)

$$\dot{y}_j(t) = G_j(x, y_j, t): \qquad \text{where } j = 1,2,\ldots\ldots,M,$$

where N components of a non-homogeneous X-force engage M components of a Y-force. As shown in Equation Set 2.1, the number of combatants of type x_i is a function of their own forces, the total Y force (the sum of the y_j forces) available to fight, and time. Similarly, the rate of change of a y_j force is a function of it's own force level, the total X force (the sum of all x_i forces) available and time.

The following set of equations represent the Moose/Wozencraft model:

$$\dot{x}_i(t) = -u_i x_i(t) - \sum_j a_{ij} x_i(t) y_j(t) - \sum_j b_{ij} y_j(t) + r_i \quad \text{where } i = 1,2,\ldots\ldots,N,$$

(eqn 2.2)

$$\dot{y}_j(t) = -v_j y_j(t) - \sum_i c_{ij} x_i(t) y_j(t) - \sum_i d_{ij} x_i(t) + s_j \quad \text{where } j = 1,2,\ldots\ldots,M,$$

where

1) The x_i and y_j represent Blue and Orange forces of different types
2) The u_i and v_j are self attrition coefficients
3) The a_{ij} and c_{ij} are area fire coefficients
4) The b_{ij} and d_{ij} are aimed fire coefficients
5) The r_i and s_j are resupply coefficients
6) $x_i, y_j \geq 0$ for all i and j
7) $\sum_i x_i = X, \sum_j y_j = Y.$

The M/W model is designed as an NxM, heterogeneous force, combat attrition model. The significance of this fact is that it allows for closer modeling of the modern combat force discussed in Chapter One.

In the M/W model, Equation Set 2.2, several things are different from the original Lanchester Equation models presented in Chapter One. One major difference is that the M/W model allows for modeling N type X forces and M type Y forces. The Moose/Wozencraft model treats the attrition of a given force as one process where Lanchester's original work broke the attrition process into two models, aimed and area fire models. With that fact in mind then, the rate of change of a force x_i is a function of four separate factors; self-attrition, area fire attrition, aimed fire attrition and resupply. In the case of determining the attrition to force x_i , the self-attrition is modeled using a self-attrition coefficient $-\mathbf{u_i}$ which is multiplied by the current x_i force level. This yields the attrition to force x_i which is attributed to 'self losses' such as disease and defections. The area fire attrition is modeled similarly to Lanchesters original model, with the added capability to account for all the possible combinations of y_j forces which can cause casualties to x_i by means of area fire. Thus an attrition-rate coefficient for each possible combination of forces engaging in battle must be available. Each y_j force has an area fire coefficients $-\mathbf{a_{ij}}$ associated with it. The sum of the attrition-rate coefficients (a_{ij}) multiplied by the y_j force level can then be thought of as an aggregate area fire attrition rate which multiplies the x_i force level. The aimed fire is very similar. It is also similar to the original Lanchester model with the added capabilities to model NxM force battles. The aimed fire is modeled by multipling each y_j force by an attrition-rate coefficient $-\mathbf{b_{ij}}$ and summing them up. This is then the aggregate aimed fire attrition rate. Finally the last factor modeled is the resupply of each x_i force with a resupply variable $\mathbf{r_i}$. The sum of the self-losses, area fire losses, aimed fire losses and resupply is equal to the change in the x_i force.

The model determines the change in a y_j force in exactly the same manner. The attrition rate coefficients are different of course, but the process is the same. The self-attrition coefficient for y_j is $-\mathbf{v_j}$. The area-fire attrition coefficients are given as $-\mathbf{c_{ij}}$ and the aimed fire coefficients are given as $-\mathbf{d_{ij}}$. The resupply variable for y_j is $\mathbf{s_j}$.

The relative usefulness of this model for modeling the combat attrition process on today's battlefield is apparent from this discussion. First, it provides the capability to model a battle involving two heterogeneous forces which we saw is necessary to analyze the optimum force allocation issue. Secondly, it is a deterministic model, so it is easy to understand and interpret. It has many similarities to the original Lanchester Equation models which are widely studied and understood.

The results of Professor Moose and Professor Wozencraft's research into the dynamical properties of this model are provided in a paper which they are submitting to the Military Operations Research Society for publication titled *Characteristic Trajectories of Generalized Lanchester Equations* [Ref. 6].

C. STOCHASTIC COMPUTER SIMULATION MODEL

The stochastic model is a Monte Carlo Simulation designed to test the Moose/Wozencraft model. The basis of this model was designed by Don E. Harrison, Jr., a professor at the Naval Postgraduate School. Professor Harrison was asked to design a stochastic model of the attrition process which included the same factors as the Moose/Wozencraft model. The combination of his efforts and some additional code to run the program for a given number of replications and calculate statistical data resulted in the model shown in Appendix B entitled Computer Simulation model (C/S).

The C/S model will be presented in the following sections:

1) Input parameters
2) The Combat Cycle
3) The Output

The material in each section is presented in a condensed users manual form to provide the reader with an understanding of the model and the capability to exercise the model if desired.

1. Input parameters

The computer code is written in FORTRAN77 and uses an exec file to define all the input files, output files, load and execute the program.

The input file contains all the required input data. Figure 2.1 shows the contents and format of the input data file. The first line of the file is a heading which is read by the program and allows the user to label and keep track of which test data set is being utilized. Next the data required for the x_i forces is given. On the second line the number of types of X forces is given (the variable NTYPX in the program), this corresponds to the N value in the M/W model which specifies the number of types of X forces. Then the initial force level for each x_i force is given, their locations, their corresponding self-attrition coefficients (u_i) and the resupply variables (r_i). Next, the attrition-rate coefficients are provided in matrix form. The aimed fire attrition-rate coefficients 'd_{ij}' are given in lines three through five. Area fire attrition-rate coefficients 'c_{ij}' are given in lines six through eight. Then the required information for the y_j forces is listed. Once again, the number of type of y_j forces is given (the NTYPY variable in the program) which corresponds to M in the M/W model. Then each y_j initial force level, location, self attrition-rate coefficients 'v_j' and resupply variables 's_j' are given. Lines ten through twelve contain the aimed fire coefficients 'b_{ij}' and lines thirteen through fifteen contain the area fire coefficients 'a_{ij}'. Line sixteen of the file contains seed numbers for the pseudo-random number generators.

1 01/16/87 TEST OF (2X2), 1 UNSTABLE ROOT S

2 NSTEP

3 NTYPX NTX1 NTX2 NTX3 loc1 loc2 loc3 u_1 u_2 u_3 r_1 r_2 r_3

4 d_{11} d_{12} d_{13}

5 d_{21} d_{22} d_{23}

6 d_{31} d_{32} d_{33}

7 c_{11} c_{12} c_{13}

8 c_{21} c_{22} c_{23}

9 c_{31} c_{32} c_{33}

10 NTYPY NTY1 NTY2 NTY3 loc1 loc2 loc3 v_1 v_2 v_3 s_1 s_2 s_3

11 b_{11} b_{12} b_{13}

12 b_{21} b_{22} b_{23}

13 b_{31} b_{32} b_{33}

14 a_{11} a_{12} a_{13}

15 a_{21} a_{22} a_{23}

16 a_{31} a_{32} a_{33}

17 DSEED DSEED DSEED

Figure 2.1 Input Data File.

The program is designed to be interactive before it begins the combat cycle which allows the user to change the heading of the input file if so desired. This also enables the user to ensure that the input file is the one desired.

2. **The Combat Cycle**

The combat cycle is a stochastic process. During the combat cycle the combatants are chosen at random and the probability of self-loss, aimed fire loss, and area fire losses are all tested by use of a pseudo-random number between 0.0 and 1.0. The cycle begins by randomly selecting which force, X or Y, will fire. Once the force has been selected then a combatant from the force is selected. For the purpose of presentation, say that $x_1(1)$ (that is combatant one of force type x_1) is selected to be the firer, then the first thing that occurs in the cycle is a check to see if combatant $x_1(1)$ is a self loss. This is accomplished by use of a random number (between 0.0 and

1.0) which is compared against the appropriate self attrition coefficient u_I. If the combatant is not a self loss then a Y force combatant is selected at random as a target, say $y_2(2)$ (the second combatant of force type y_2) as an example. Once the firer and target have been identified, the next step tests the effectiveness of the 'shot' by using a random number to test whether the target is killed. This process involves first determining if the target is killed, which is done by testing whether a random number is less than the total kill probablity (which is the sum of the aimed fire and area fire attrition-rate coefficients, or their kill probabilities). If the random number is less, then the target has been killed and if not the shot missed. If the target was killed then this same process is followed to determine whether it was a loss due to aimed fire or area fire. Required data is tabulated on the number of combatants of type y_j which are killed by type x_i forces and each force is resupplied. The combat cycle is repeated in this manner until all combatants have had an opportunity to fire during each timestep. This process is continued for each timestep until one of two conditions occurs:

1) One force reaches a break point which has been defined as NXSTOP and NYSTOP. This simulates the level of attrition a commander will suffer before pulling back or surrendering rather than fighting to the last man.
2) The combat cycle has been repeated for a given number of timesteps, which is specified in the input data file.

3. The Output

The C/S model was designed to provide several output files for data analysis. The model generates two files which provide a very detailed timestep-by-timestep recording of the combat cycle outcome. These files require a great deal of storage space and therefore during the interactive portion of the program the user is asked if they want these files printed in their complete form. It is not recommended that they be printed if the user desires multiple replications because the storage space required is too large. One of these files provides a detailed listing of each timestep results including a summary of the force levels, the number of losses by each type of fire and the number of combatants that were resupplied. An example of this file's output is shown in Figure 2.2. The other file contains similar listings with the number of dead combatants of each type provided as well. An example of this file's output is shown in Figure 2.3.

The remainder of the files are designed to provide the force levels of each force (i.e., x_I and y_I) for each timestep. This provides the data which can be used to plot

```
02/02/87  TEST OF (3X3),
# X TYPES  NUMBER EACH    LOCATIONS      SELF-LOSS       RE-SUPPLY
     3  200  150  175  1.0  1.0  1.0  0.005  0.010  0.020    4    4   11

KILL PROBABILITIES FOR FORCE X(I) SHOOTS Y(J)

PK(1,1) PK(1,2) PK(1,3) PK(2,1) PK(2,2) PK(2,3) PK(3,1) PK(3,2) PK(3,3)
0.06000 0.02000 0.02000 0.00000 0.00000 0.00000 0.03000 0.01000 0.02000


AREA FIRE COEFFICIENTS FORCE X(I) SHOOTS Y(J)

CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3) CC(3,1) CC(3,2) CC(3,3)
0.00000 0.00000 0.00000 0.00000 0.00030 0.00020 0.00000 0.00000 0.00010

# Y TYPES  NUMBER EACH    LOCATIONS      SELF-LOSS       RE-SUPPLY
     3  150  125  225  1.0  1.0  1.0  0.003  0.020  0.030    4    5   14

KILL PROBABILITIES FOR FORCE Y(I) SHOOTS X(J)

PK(1,1) PK(1,2) PK(1,3) PK(2,1) PK(2,2) PK(2,3) PK(3,1) PK(3,2) PK(3,3)
0.05000 0.01000 0.05000 0.00000 0.00000 0.00000 0.02000 0.01000 0.03000


AREA FIRE COEFFICIENTS FORCE Y(I) SHOOTS X(J)

AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3) AA(3,1) AA(3,2) AA(3,3)
0.00000 0.00000 0.00000 0.00000 0.00020 0.00030 0.00000 0.00000 0.00010

     DOUBLE PRECISION RANDOM VARIANT SEEDS          INTEGER SEED VALUES
0.345215790D+09 0.456357610D+08 0.893427610D+08 1209308051 1194034571 1196766258

#    NTXS  NTX  NX1  NX2  NX3 RX1 RX2 RX3 NTYS  NTY  NY1  NY2  NY3 RY1 RY2 RY3
+   0  525  525  200  150  175   4   4  11  500  500  150  125  225   4   5  14


TIMESTEP   1
                   AIMED FIRE KILLS OF Y"S BY X FORCE

  D(1,1)  D(1,2)  D(1,3)  D(2,1)  D(2,2)  D(2,3)  D(3,1)  D(3,2)  D(3,3)
              3       0       2       0       0       0       1       1       0

                   AIMED FIRE KILLS OF X"S BY Y FORCE

  B(1,1)  B(1,2)  B(1,3)  B(2,1)  B(2,2)  B(2,3)  B(3,1)  B(3,2)  B(3,3)
              2       1       1       0       0       0       2       0       2

                   AREA FIRE KILLS OF Y"S  BY X FORCE

  C(1,1)  C(1,2)  C(1,3)  C(2,1)  C(2,2)  C(2,3)  C(3,1)  C(3,2)  C(3,3)
              0       0       0       0       5       7       0       0       2

                   AREA FIRE KILLS OF X"S  BY Y FORCE

  A(1,1)  A(1,2)  A(1,3)  A(2,1)  A(2,2)  A(2,3)  A(3,1)  A(3,2)  A(3,3)
              0       0       0       0       2       5       0       0       3

  # NTXS  NTX  NX1  NX2  NX3 DX1 DX2 DX3 NTYS  NTY  NY1  NY2  NY3 DY1 DY2 DY3
+   1  525  525  200  150  175   5   5  13  500  500  150  125  225   4   9  20
```

Figure 2.2 Example File 6 Output.

```
TOTAL X FORCE LOSS WAS  23, RESUPPLY WAS:   4   4  11
     AIMED FIRE LOSSES:   4   1   3
       AREA FIRE LOSSES:   0   2   8
              SELF LOSSES:   1   2   2
TOTAL Y FORCE LOSS WAS  33, RESUPPLY WAS:   4   5  14
     AIMED FIRE LOSSES:   4   1   2
       AREA FIRE LOSSES:   0   5   9
              SELF LOSSES:   0   3   9
+  1  525  521  199  149  173   4   4  11  500  490  150  121  219   4   5  14
                    AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE

AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3) AA(3,1) AA(3,2) AA(3,3)
 0.00000 0.00000 0.00000 0.00000 0.00011 0.00023 0.00000 0.00000 0.00008

                    AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE

BB(1,1) BB(1,2) BB(1,3) BB(2,1) BB(2,2) BB(2,3) BB(3,1) BB(3,2) BB(3,3)
 0.03500 0.02333 0.02000 0.00000 0.00000 0.00000 0.02333 0.00000 0.02667

                    AVERAGE ATTRITION COEFFICIENTS FOR X FORCE

CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3) CC(3,1) CC(3,2) CC(3,3)
 0.00000 0.00000 0.00000 0.00000 0.00027 0.00021 0.00000 0.00000 0.00005

                    AVERAGE ATTRITION COEFFICIENTS FOR X FORCE

DD(1,1) DD(1,2) DD(1,3) DD(2,1) DD(2,2) DD(2,3) DD(3,1) DD(3,2) DD(3,3)
 0.05000 0.00000 0.02222 0.00000 0.00000 0.00000 0.01905 0.02286 0.00000

STOPPED BY PROGRAM AT NSTOP =      1
```

Figure 2.2 Example File 6 Output. (cont'd.)

force level trajectories, which allows for ease in performing a comparison of the two model's results for a given input set.

Now that each of the models has been presented, chapter three will discuss the design of the experiment, and provide samples of the output files used for plotting the force level trajectories.

```
 02/02/87  TEST OF (3X3),
   # NTXS  NTX  NX1  NX2  NX3 DX1 DX2 DX3 NTYS  NTY  NY1  NY2  NY3 DY1 DY2 DY3
 *   0  525  525  200  150  175   0   0   0  500  500  150  125  225   0   0   0

   D,B,C,A BY ROWS IN DESCENDING ORDER
     1       3       0       2       0       0       0       1       1       0
     1       2       1       1       0       0       0       2       0       2
     1       0       0       0       0       5       7       0       0       2
     1       0       0       0       0       2       5       0       0       3
 TOTAL X FORCE LOSS WAS  23, RESUPPLY WAS:   4   4  11
     AIMED FIRE LOSSES:   4   1   3
       AREA FIRE LOSSES:   0   2   8
              SELF LOSSES:   1   2   2
 TOTAL Y FORCE LOSS WAS  33, RESUPPLY WAS:   4   5  14
     AIMED FIRE LOSSES:   4   1   2
       AREA FIRE LOSSES:   0   5   9
              SELF LOSSES:   0   3   9
 *   1  525  521  199  149  173   5   5  13  500  490  150  121  219   4   9  20

   A,B,C,D BY ROWS IN DESCENDING ORDER
 AV  0.00000 0.00000 0.00000 0.00000 0.00011 0.00023 0.00000 0.00000 0.00008
 AV  0.03500 0.02333 0.02000 0.00000 0.00000 0.00000 0.02333 0.00000 0.02667
 AV  0.00000 0.00000 0.00000 0.00000 0.00027 0.00021 0.00000 0.00000 0.00005
 AV  0.05000 0.00000 0.02222 0.00000 0.00000 0.00000 0.01905 0.02286 0.00000
 STOPPED BY PROGRAM AT NSTOP =      1
```

Figure 2.3 Example File 7 Output.

III. RESEARCH METHODOLOGY

A. OBJECTIVE

The primary objective of this research is to determine whether a deterministic combat attrition model is a good approximation for the mean result of a stochastic model of the combat attrition process. With that objective in mind, an experiment was designed to preform a comparison of two such models. The models used in this experiment are the Moose Wozencraft model (M W model), which is a deterministic model, and a Monte Carlo Simulation (C S model) which were introduced in chapter two. A secondary objective is to see if the two models behave with similar characteristics as the results of Professor Moose and Professor Wozencraft's research [Ref. 6].

B. EXPERIMENTAL DESIGN

The experiment is designed to run the C S model for ninety-nine timesteps over thirty replications, to generate force level data and calculate the resulting average attrition rate coefficients. These average coefficients are then used as the input parameters to the M W model which then provides the calculated force levels per timestep. This force level data is used for analysis. The method for analysis which has been chosen is to plot the resulting force level data over time to provide for an easier comparison than sorting through the enormous amount of data generated for each test case.

This experiment was conducted for cases involving 1x1, 1x2, 2x2, and 3x3 force level scenarios. Examples of the data collected and the resulting force level trajectory plots are provided in the next section.

1. Model Verification

Before beginning the experiment using these two models, each of the models was exercised and tested heavily to verify that they ran properly and provided the desired output as they were designed. A brief description of the verification process for each model will be given.

a. Computer Simulation Model (C/S model)

The first step in verifying the C S model involved a procedure of several runs using different input data sets to test the program to ensure that

1) The input data and program were compatible and the input variables are properly read.

2) All the computational code was performing properly (i.e., the average attrition rate coefficients and the average force levels).

3) That each of the test conditions was functioning as designed (i.e., the condition for ending at the specified force levels XSTOP and YSTOP).

4) That the output was presented as desired for the step-by-step analysis and the plotting data.

5) Test the resulting average attrition-rate coefficients with the input probabilities of kill which correspond to these coefficients.

During this portion of the research it was interesting to note that the C/S model was designed to allow each shooter one shot per time interval, but the shooter could select a target to fire at which had already been killed during that time interval. This fact seemed to be a fairly realistic condition that could occur on the battle field where two combatants fire at the same target instantaneously or within a fraction of a second of each other. Therefore, this condition was left in the model.

b. Moose/Wozencraft Model (M/W model)

A computer program was designed to run the M/W model to allow for an easy comparison of the two models. By utilizing a computer program it allowed for the same timestep by timestep analysis as the C/S model.

After designing the FORTRAN program, shown in Appendix D, to run the Moose/Wozencraft model (M/W model) it was tested to verify that the following conditions were met:

1) The input data and the program were compatible and all variables were read properly.

2) All computational code was performing as designed.

3) The resulting output files provided the desired data for analysis. Since the M/W model is a deterministic model, the same output will be obtained each time for the same set of input data parameters.

Each of the conditions listed above was verified for simple input data sets for each of the cases.

2. Data Generation Procedure

The procedure followed in conducting this experiment was a three step process for each unique case tested. The first step involved running the C/S model for ninety-nine timesteps over thirty replications which would generate the following data files:

1) The calculated average attrition-rate coefficients which are later used as input to the M W model. An example file is shown in Figure 3.1.

```
01/27/87  TEST OF (1X1)

                    AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE

AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3) AA(3,1) AA(3,2) AA(3,3)
0.00012 0.00000 0.00000 0.00001 0.00000 0.00000 0.00000 0.00000 0.00000

                    AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE

BB(1,1) BB(1,2) BB(1,3) BB(2,1) BB(2,2) BB(2,3) BB(3,1) BB(3,2) BB(3,3)
0.04857 0.00000 0.00000 0.00142 0.00000 0.00000 0.00000 0.00000 0.00000

                    AVERAGE ATTRITION COEFFICIENTS FOR X FORCE

CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3) CC(3,1) CC(3,2) CC(3,3)
0.00027 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000

                    AVERAGE ATTRITION COEFFICIENTS FOR X FORCE

DD(1,1) DD(1,2) DD(1,3) DD(2,1) DD(2,2) DD(2,3) DD(3,1) DD(3,2) DD(3,3)
0.00648 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000

STOPPED BY PROGRAM AT NSTOP =      99
```

Figure 3.1 Example Average Attrition-rate Coefficients File For C S Model

2) A file containing the average force level for each force, the average force level plus one standard deviation, and the average force level minus one standard deviation is provided for the aggregate force levels X ($\sum x_i$) and Y ($\sum y_j$) per timestep as well as for each of the individual units x_i and y_j. An example file is shown in Figure 3.2.

3) The average force level and standard deviation of each force level per time step. An example of these results is shown in Figure 3.3.

The second step involves using the average attrition-rate coefficients calculated by the C S model as input parameters to the M W model. The output generated by the M W model for each set of input data is the following:

1) The force level for each of the forces x_i and y_j as well as the total force levels X ($\sum x_i$) and Y ($\sum y_j$) for each time step. An example of this output file is shown in Figure 3.4.

01 16 87 TEST OF (2X2), 1 UNSTABLE ROOT

T	x_1	y_1	$x_1+\sigma$	$y_1+\sigma$	$x_1-\sigma$	$y_1-\sigma$
1	203.8	313.9	209.2	320.2	198.3	307.6
2	207.1	322.0	213.8	331.5	200.4	312.4
3	210.7	327.4	219.1	336.9	202.3	317.9
4	212.0	329.3	223.0	340.1	201.0	318.4
5	215.2	327.8	225.7	340.4	204.7	315.2
. . .						
97	250.1	284.8	259.8	296.2	240.3	273.4
98	251.3	283.7	261.9	294.7	240.7	272.8
99	252.5	283.0	262.9	294.9	242.1	271.2

Figure 3.2 Example Average x_1y_1 Force Level File For C S Model

01 27 87 TEST OF (1X2), CASE 5

T	x_1	σ_{x1}	y_1	σ_{y1}
1	493.43	6.64	170.37	5.16
2	492.27	9.54	149.47	6.24
3	496.23	11.75	132.63	6.63
4	502.47	12.29	119.67	6.35
5	510.30	16.21	109.67	6.38
. . .				
95	939.63	21.95	67.10	4.42
96	939.80	25.08	67.20	4.43
97	940.13	26.22	68.20	4.37
98	941.07	25.40	68.77	4.44
99	942.87	26.01	68.43	4.14

Figure 3.3 Example File of Force Level and Standard Deviation For C S Model.

The third step involves plotting the force level data generated by each of the models to perform the comparison. The plots which were utilized for the comparison are listed below with examples provided:

1) A plot of the total X force vs. Total Y force level trajectories for both models is done first. This plot provides for a macro level comparison of the resulting

01 27 87 TEST OF (1x2), CASE 4

T	x_1	y_1
0	200.00	300.00
1	196.29	299.19
2	192.88	298.75
3	189.73	298.61
4	186.82	298.76
5	184.12	299.13
. . .	. . .	. . .
95	91.69	472.51
96	90.92	474.95
97	90.14	477.41
98	89.37	479.89
99	88.60	482.39

Figure 3.4 Example x_2y_2 Force Level Data File For M-W Model.

force levels over time (i.e., the Force Commanders level). This plot is two dimensional, plotting the X force level vs. Y force level. The time variable is shown by marking the force level every fifth timestep. This plot will be used to analyze the two aggregate force models results. An example of the total force level trajectory plot is shown in Figure 3.5.

2) Plots of the x_i force level vs. time and the y_j force levels vs. time for both the C-S and the M-W models are given. This provides a comparison of the battle outcome for the two models at the micro level. An example of the x_i (or y_j) plot is shown in Figure 3.6.

The plots introduced in this chapter are used extensively for conducting the comparison of the two models. Chapter Four will present the results of the experiment and discuss the comparisons.

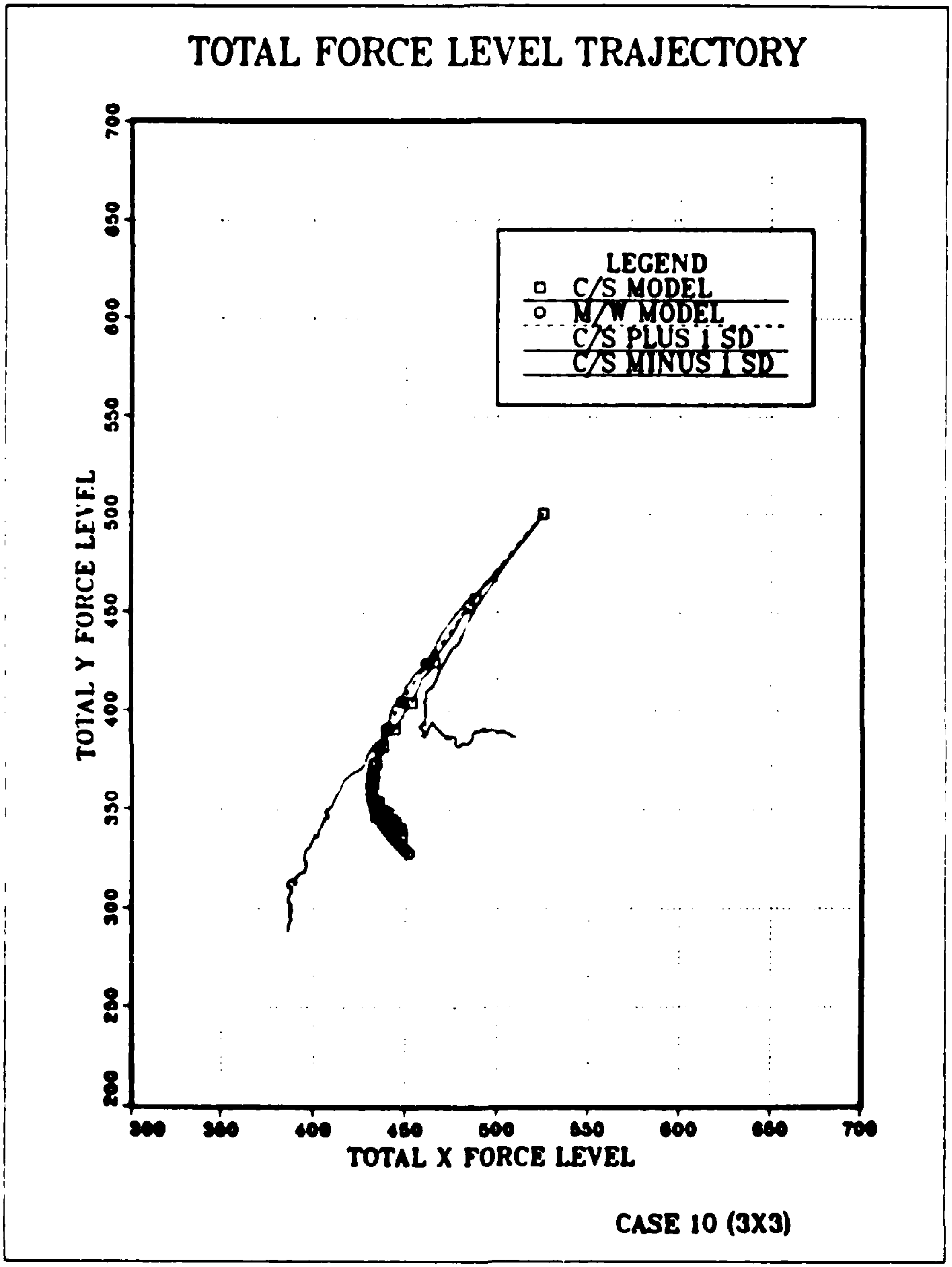

Figure 3.5 Example Total Force Level Trajectory Plot.

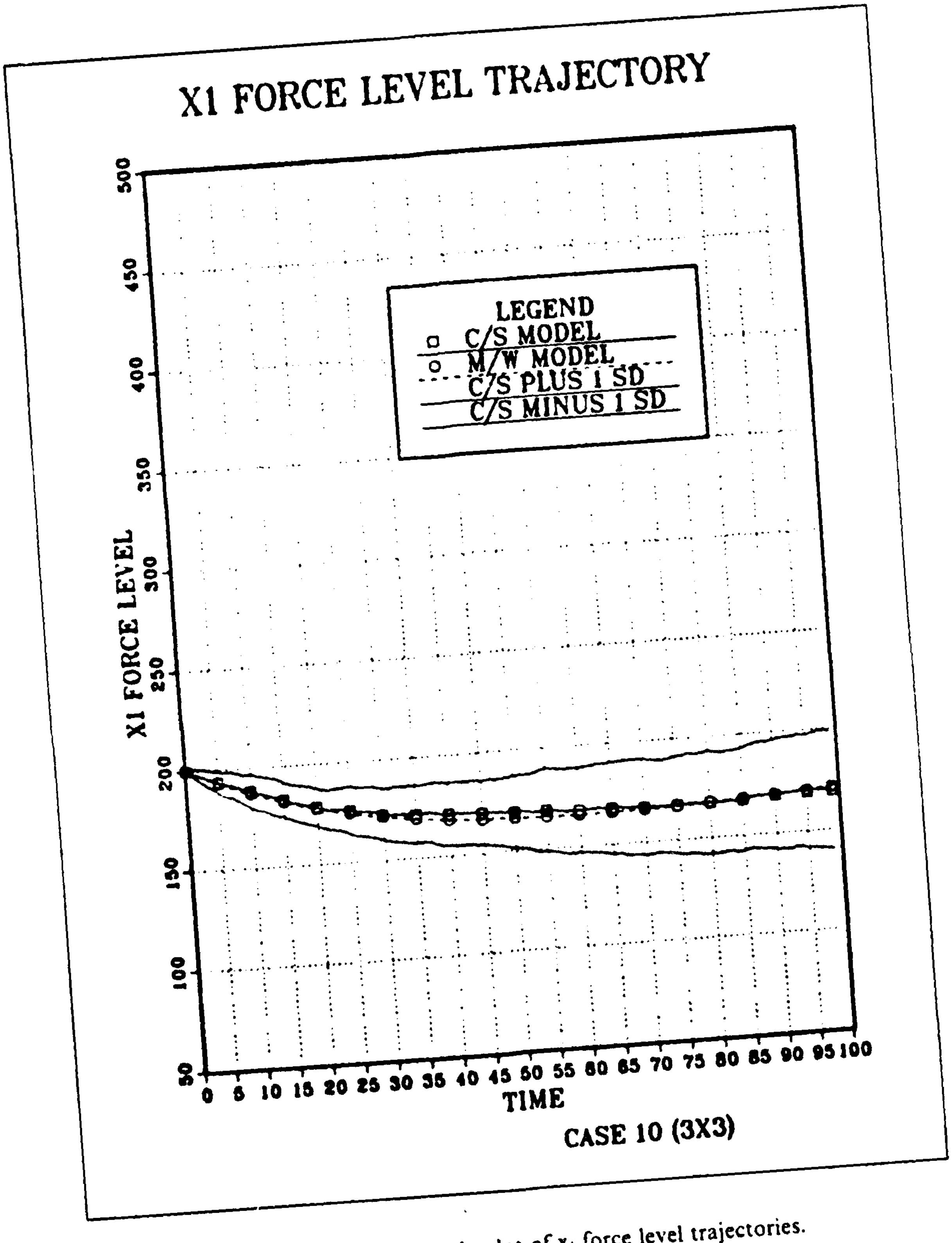

Figure 3.6 Example plot of x_i force level trajectories.

IV. EXPERIMENTAL RESULTS

A. INTRODUCTION

The primary objective of this experiment was to test whether the deterministic Moose/Wozencraft model was a good approximation (or prediction) for the mean result of the stochastic computer simulation. The secondary objective for this experiment was to test whether the two models' force level trajectories behaved in the expected manner described by Professors Moose and Wozencraft. The results of ten cases are presented graphically showing the two models' results with a bracket of plus and minus one standard deviation as a measure of uncertainty. Each case will be discussed in the context of the primary and secondary objectives in this chapter and their results shown graphically in Appendix E.

B. THE FOUR MODES OF BEHAVIOR

Before proceeding with the presentation of the results, it is important to briefly explain the four trajectory behavior modes which will be referenced in each of the cases. Professors Moose and Wozencraft found that the force level trajectories displayed one of four distinct modes of behavior. A mode of behavior was the result of the existence, or nonexistence, of equilibrium points in the state space (positive first quadrant). The four modes related to the existence of one stable equilibrium, one unstable equilibrium, two equilibria (where one is stable and one is unstable), or no equilibria. These four modes of behavior are illustrated in Figure 4.1. [Ref. 6]

The expected behavior for any force level trajectory when one stable equilibrium point exists in the state space is shown in Figure 4.1 (a). The trajectory will be attracted to the stable equilibrium and remain there, since $dx/dt = dy/dt = 0$ at the stable equilibrium.

The expected behavior for any force level trajectory when one unstable equilibrium point exists in the state space is shown in Figure 4.1 (b). As the trajectories near the equilibrium they are repelled or turned away from the equilibrium point. The presence of an unstable equilibrium creates a division of the state space into two regions. The dividing line was called a separatrix. In scenarios which exhibit this mode of behavior the outcome depends on which side of the separatrix the initial force levels begin.

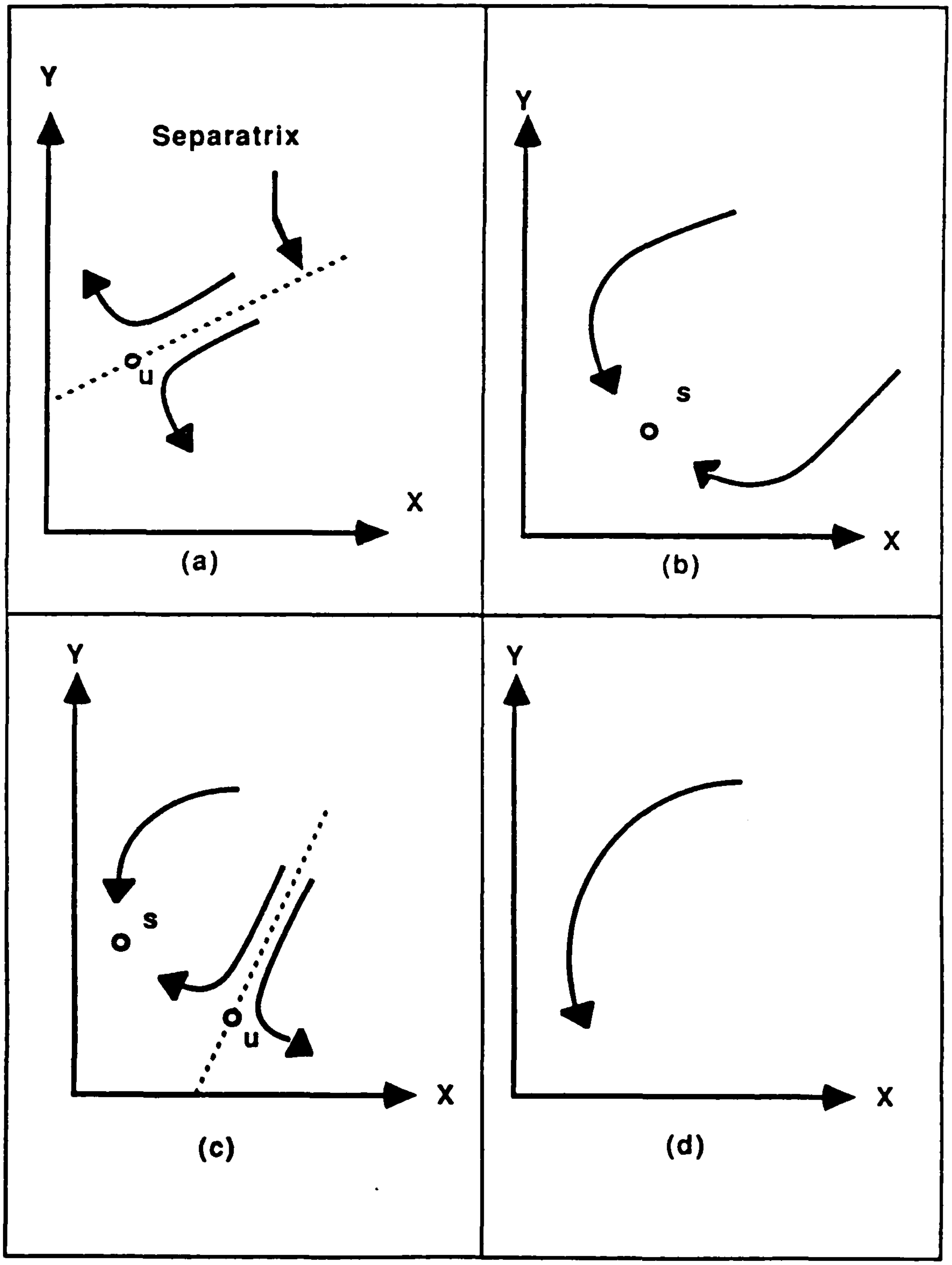

Figure 4.1 The Four Trajectory Modes Of Behavior.

The expected behavior for any force level trajectory when two equilibria exist (one stable and one unstable) is shown in Figure 4.1 (c). Once again the unstable equilibrium creates a separatrix dividing the state space into regions. The stable equilibrium will attract trajectories as discussed earlier. This particular mode of behavior has two possible outcomes for a battle, one of the forces can win or they will fight to a draw. The outcome depends on the initial force levels for each force.

The expected behavior for any force level trajectory when no equilibrium point exists is shown in Figure 4.1 (d). The trajectories will approach, and eventually cross one of the axes, which results in a force being eliminated.

C. DISCUSSION

This numerical experiment was designed to test a combat scenario involving different opposing-force composition. (N_x x M_y) cases involving N_x types of X forces vs. M_y types of Y forces are tested in this experiment. An example is a 1x2 case which involves one X type force vs. two Y type forces. The input data sets were chosen to test the two models for the four distinct modes of behavior that Professors Moose and Wozencraft identified [Ref. 6]. The computed results are presented in sections which are dedicated to a specific (N x M) scenario.

Appendix E contains the input data sets used for each model, and the resulting force level trajectory plots for each of the cases. In each case the input data for the C/S model was developed to test one of the modes of behavior and the M/W model input data were the calculated attrition rate coefficients calculated by the C/S model. Several trajectory plots were done for each of the cases. There is a plot of the total aggregate force level trajectory which compares the total X force vs. the total Y force. This aggregate trajectory shows the battle outcome. An example of this plot is shown for case six in Figure 4.2.

The result of the computer simulation model, which is the mean of thirty replications, is always shown as a solid line with squares marking the force level every fifth time step. The Moose/Wozencraft model results are shown as a dashed line with circles marking the force level every fifth time step. Additionally, two solid lines are shown with no markings. These are the curves which show plus and minus one standard deviation from the C/S model results. They serve as a measure of uncertainty for the comparison. The aggregate force level plot provides a comparison of the two models total force level results to determine whether they do have the same general

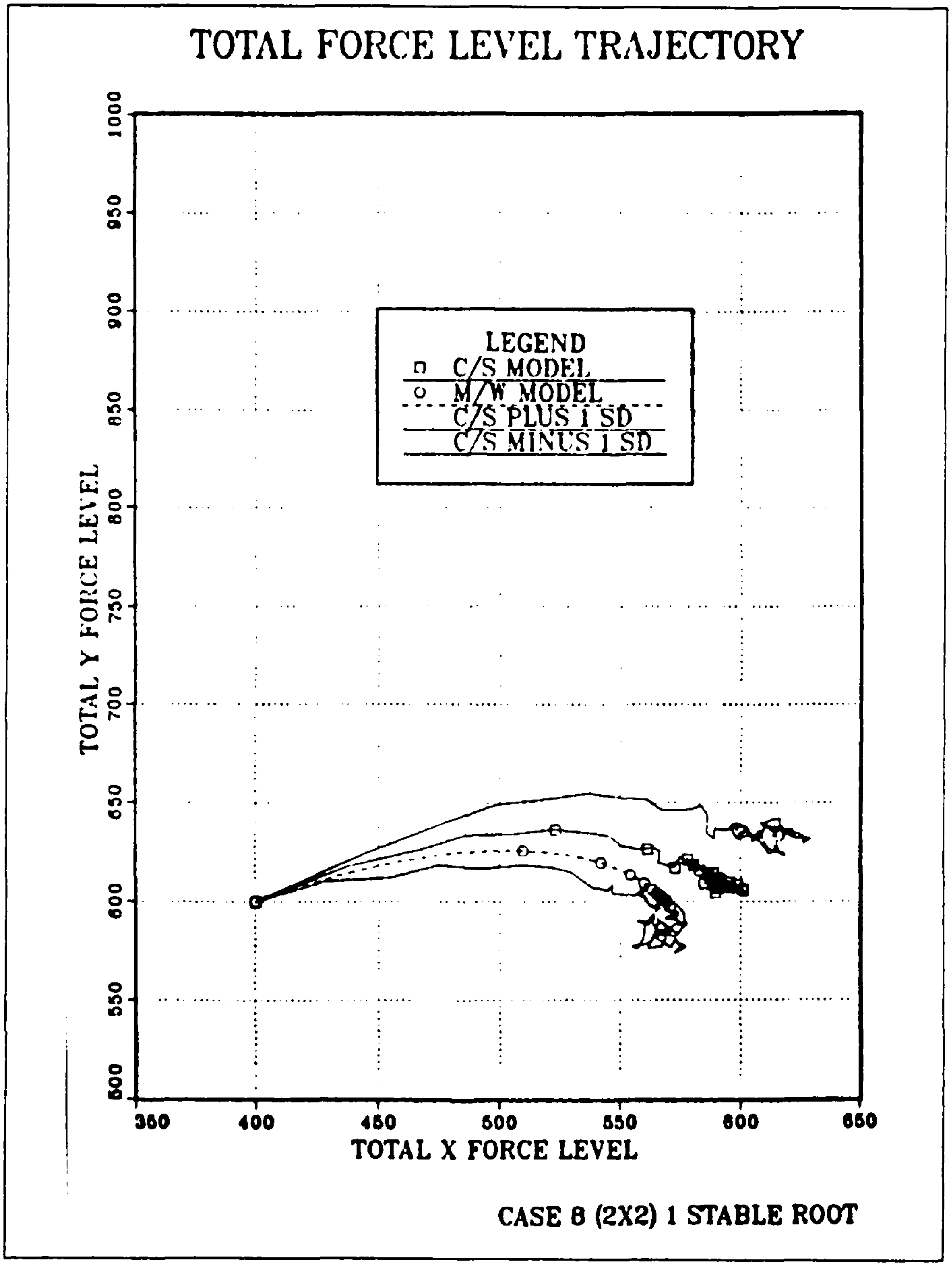

Figure 4.2 Total Force Level Trajectory For Case Eight.

behavior and whether the deterministic model is a good approximation for the result of the stochastic model. Each of the aggregate forces (i.e., $X = \sum_i x_i$) are plotted as a function of time to provide a clearer comparison of the two models results as the battle progresses. An example of these plots are shown in Figures 4.3 and 4.4. This provides another comparison between the two models' results for each aggregate force. In addition to the aggregate force level plots, the results for each of the separate units are plotted as a function of time. Examples of these plots are provided in Figures 4.5 through 4.8. This provides a comparison of the two models at the individual unit level.

The method of comparison used for this analysis was a graphical presentation of the force level trajectories rather than an extensive numerical analysis of the resulting data. Each case will be presented in a similar manner. The cases will be briefly described and then a list of observations given. Specific comments are made about whether the M/W trajectory was close to the C/S trajectory and whether the expected behavior was observed.

D. (1 X 1) CASE COMPARISONS

The experiment was conducted for cases one through four using one X force and one Y force, which are labeled as 1x1 cases. The results of each of these cases are shown in Figures E.1 through E.12 in Appendix E. The force level trajectories for the M/W model were very close to the mean force level trajectories for the C/S model in all four cases.

1. Case One

Case one was designed to test the two models involving a 1x1 scenario for the mode of behavior where no equilibrium exists. The following observations are noted from the results shown in Figures E.1 through E.3:

a) The resulting force level trajectories for the M/W model and the C/S model were very similar, both displaying the same trajectory shape.

b) The M/W model trajectory was well within the one standard deviation boundaries, for the complete battle. In fact, the two curves over lapped during portions of the battle. This case resulted in the X force winning the battle.

2. Case Two

Case two was designed to test the mode when one stable equilibrium exists in the vicinity of (200,200). The following observations are noted from the results shown in Figures E.4 through E.6:

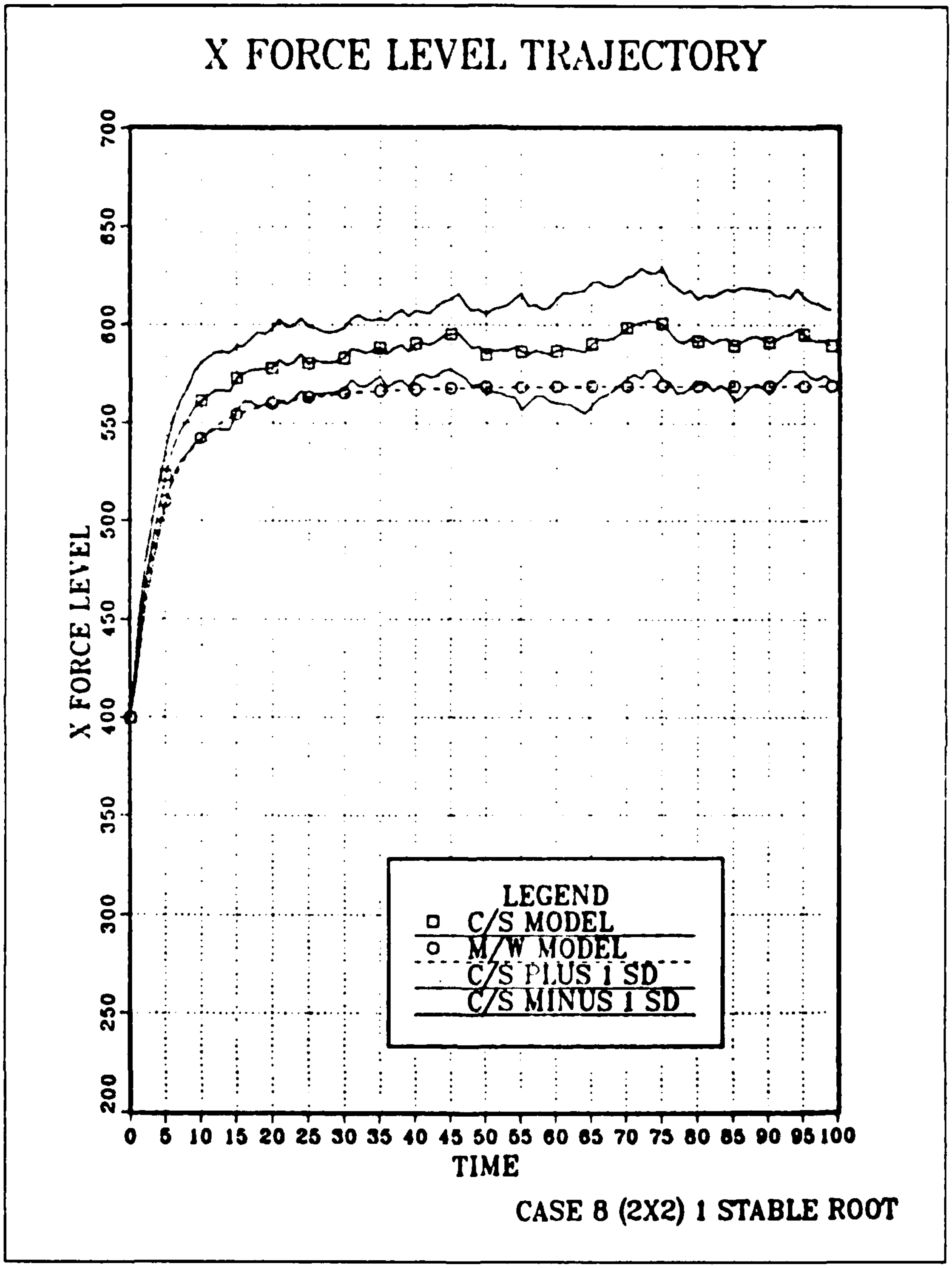

Figure 4.3 X Force Level Trajectory Over Time For Case Eight.

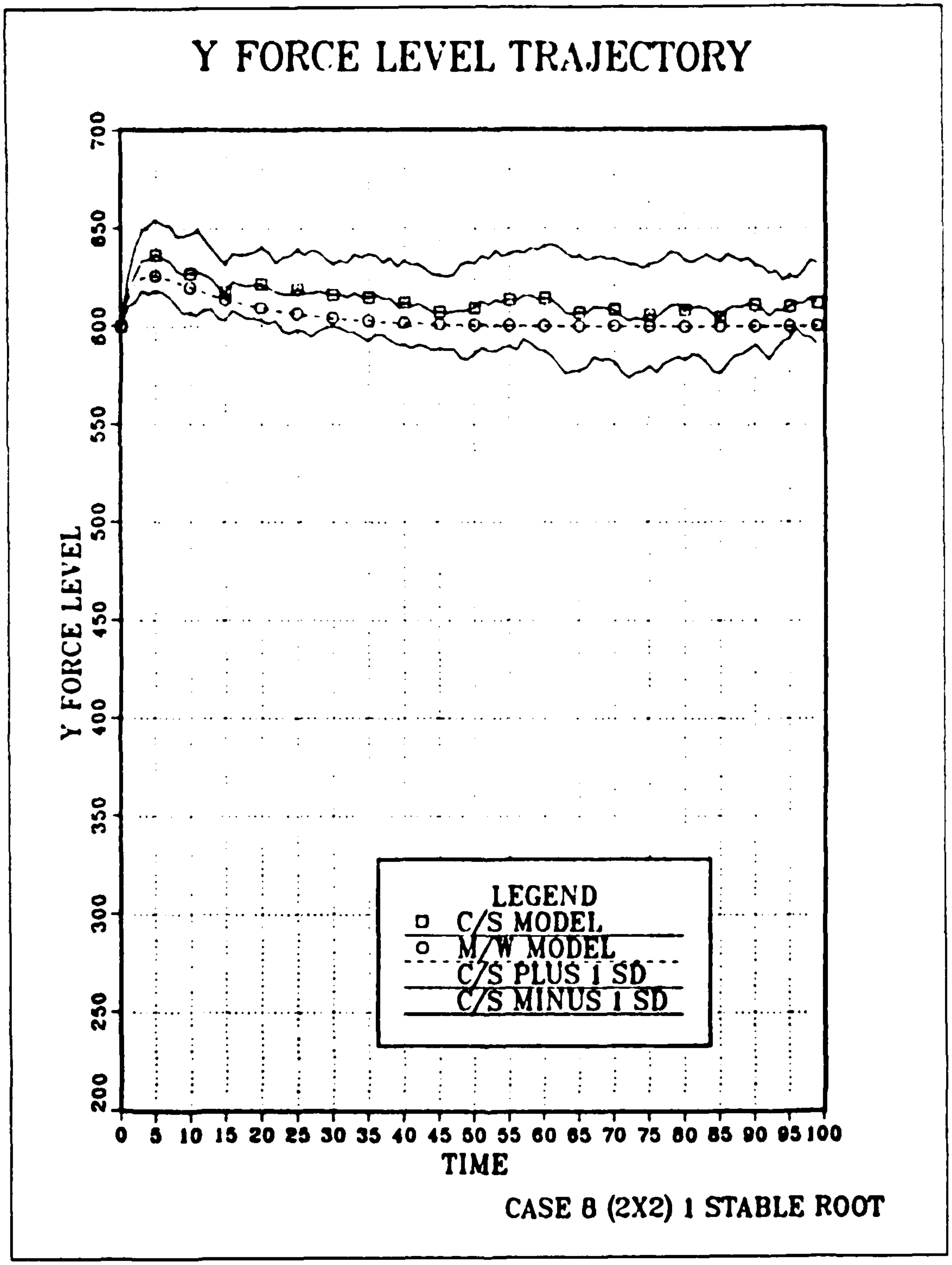

Figure 4.4 Y Force Level Trajectory Over Time For Case Eight.

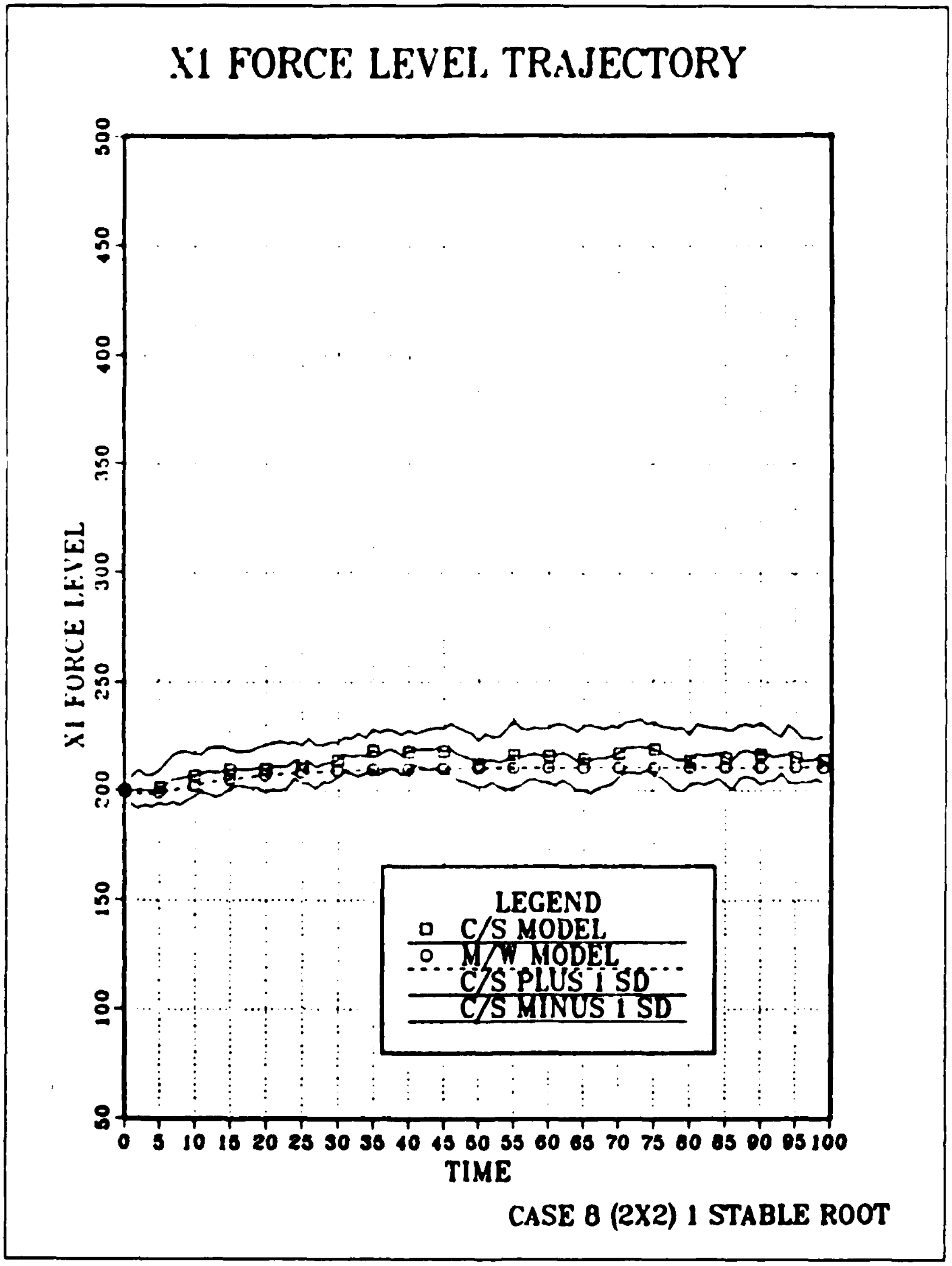

Figure 4.5 X1 Force Level Trajectory Over Time For Case Eight.

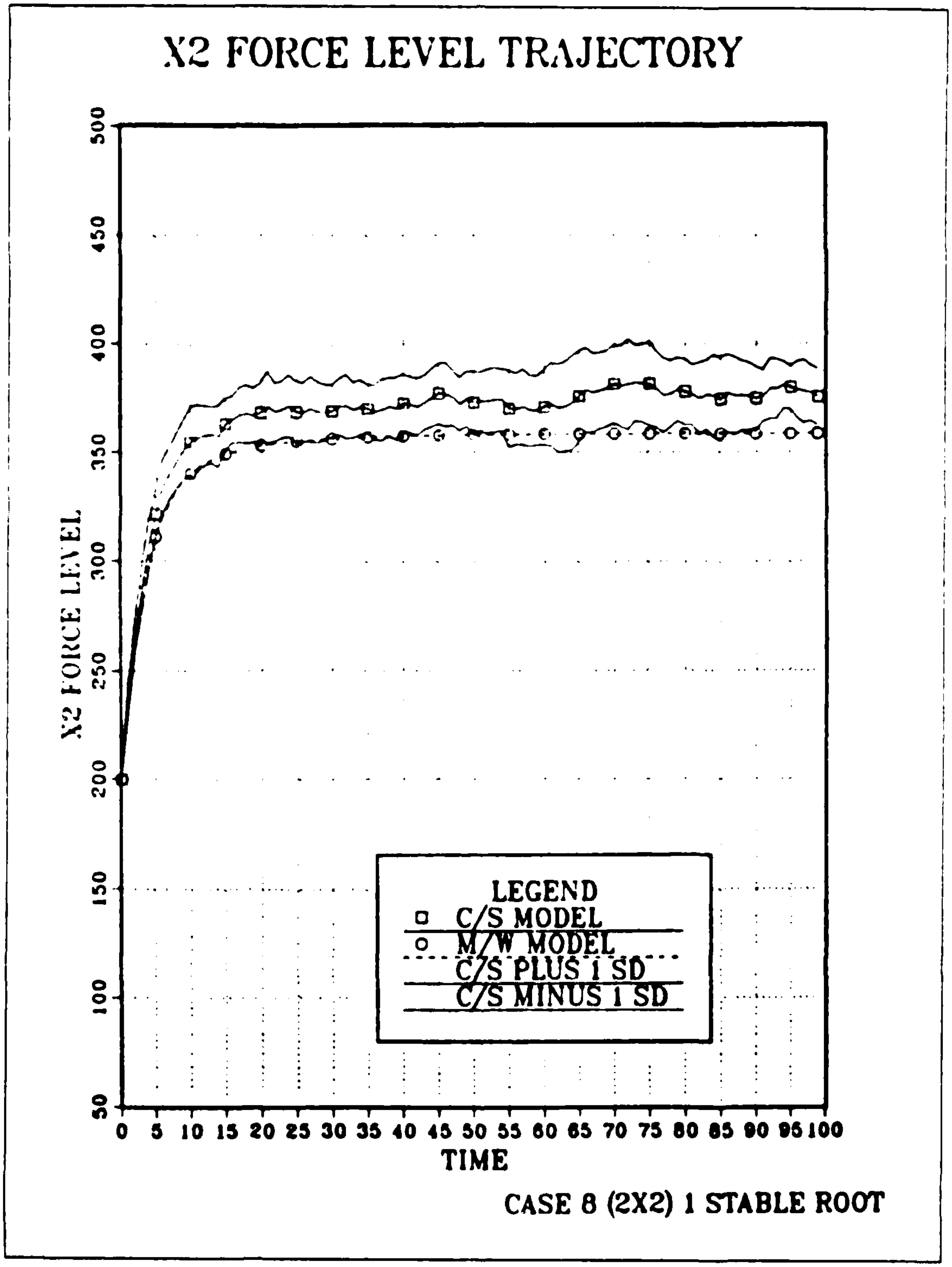

Figure 4.6 X2 Force Level Trajectory Over Time For Case Eight.

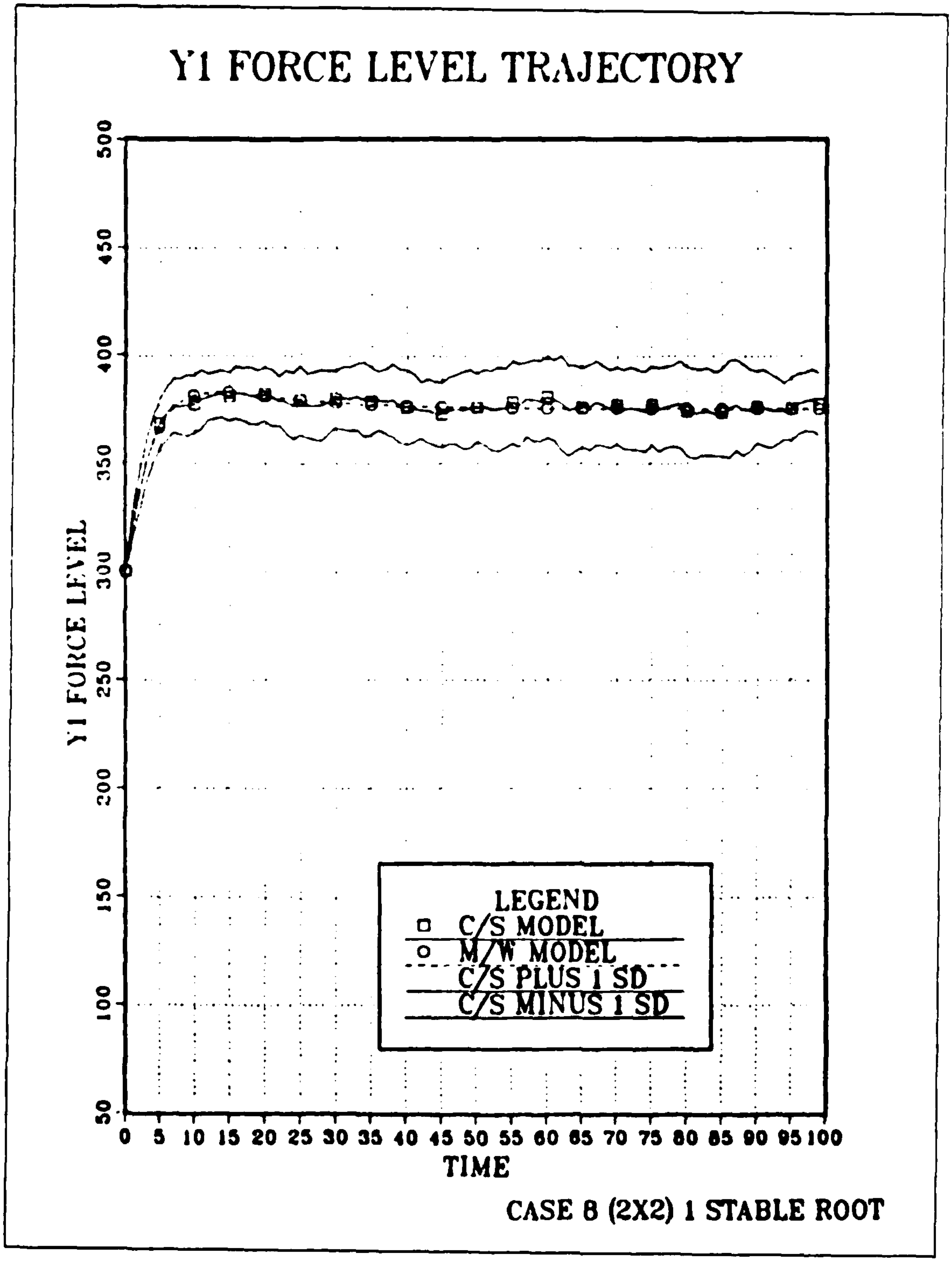

Figure 4.7 Y1 Force Level Trajectory Over Time For Case Eight.

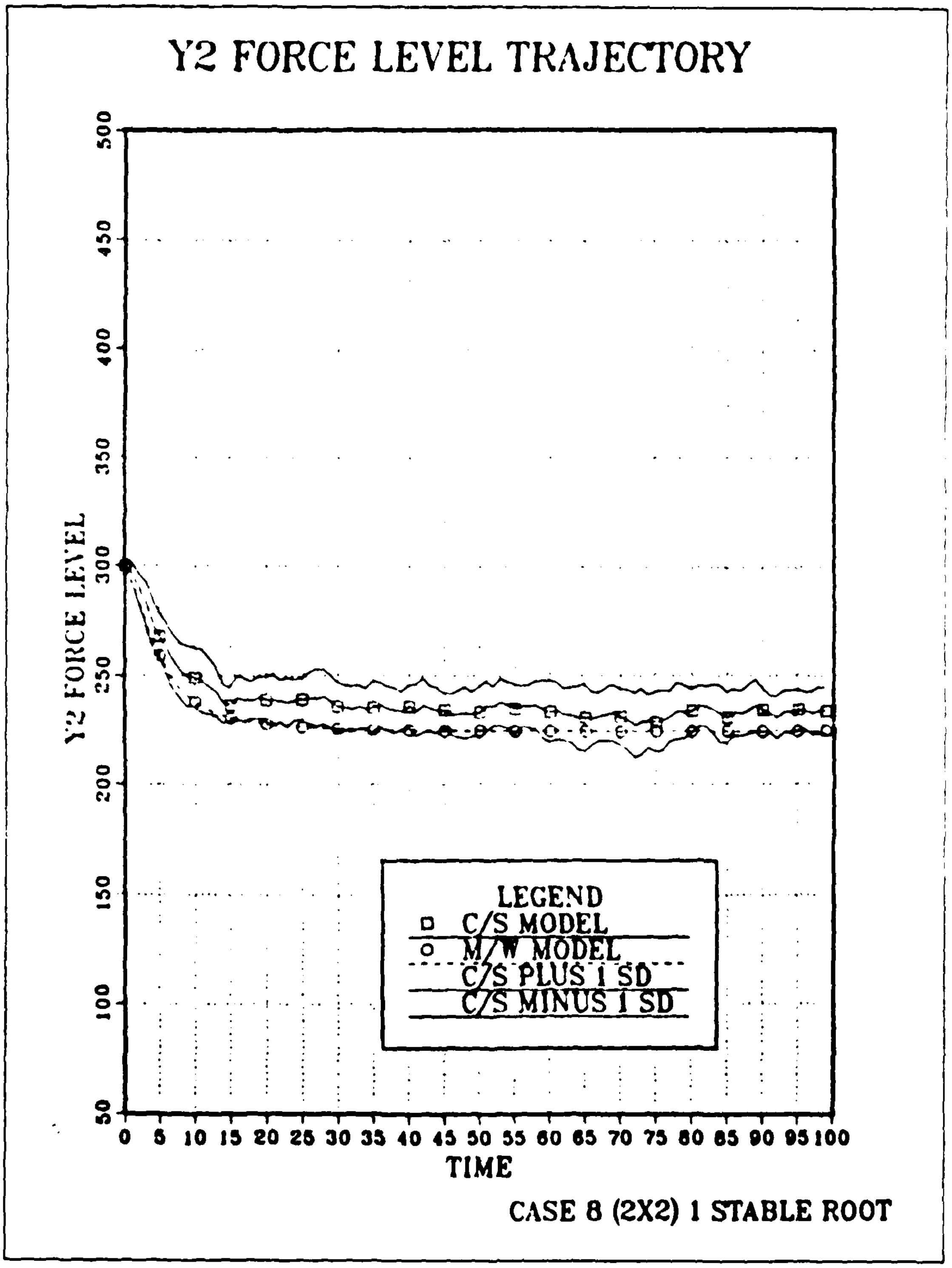

Figure 4.8 Y2 Force Level Trajectory Over Time For Case Eight.

a) The M/W trajectory and the C/S trajectory were very similar and they display the same trajectory shape.

b) The M/W model trajectory was outside the established boundaries for the initial portion of the battle. It is noted, however, that the area bounded by one standard deviation is extremely small in this region. The M/W trajectory entered the bounded area as the two curves turned. Both models reached an equilibrium point in the vicinity of (200,200) where the battle would end in a draw.

c) Examination of the individual unit force level trajectories in Figures E.5 and E.6 shows that the M/W trajectory was within the established bounds for each force as a function of time.

3. **Case Three**

Case three was designed to test the 1x1 scenario for the mode of behavior where one unstable equilibrium exists in the vicinity of (200,200). The following observations are noted from the results shown in Figures E.7 through E.9:

a) The force level trajectories of both models displayed the expected behavior for a case where one unstable equilibrium exists. Each of the models trajectories approached the equilibrium and then turn away from it.

b) Once again the M/W trajectory was well within one standard deviation of the C/S trajectory.

4. **Case Four**

Case four was designed to test a 1x1 scenario where the mode of behavior involves two equilibria in the vicinity of (200,200)(U) and (366,100)(S). The following observations are noted from the results of this case shown in Figures E.10 through E.12:

a) The results show the trajectories of both models display the expected behavior approaching the unstable equilibrium and then turning away.

b) The M/W model was always well within the one standard deviation boundaries for this case.

E. (1X2) CASE COMPARISONS

One case was developed to test an unbalanced force level scenario involving one type X force and two type Y force. The purpose of this case was to ensure that both models perform properly for an unbalanced number of forces engaging in the battle. A 1x2 scenario was chosen for this case and the results are presented next.

1. Case Five

Case five was the one unbalanced scenario developed and the mode of behavior tested involved one unstable equilibrium in the vicinity of (300,200). The following observations were noted from the results shown in Figures E.13 through E.18:

a) Both models displayed the expected behavior for an unstable equilibrium. The trajectories for both models approached the equilibrium point and then were turned away.

b) The M W trajectory was well within the one standard deviation boundaries, and had the same shape as the C S trajectory. It is interesting to note the relative speed at which the two trajectories moved away from the unstable equilibrium point. The C S model moved away much faster than the M W model.

c) The plot of each force vs. time, shown in Figures E.14 through E.18, show the two models results separating over time, and the M W trajectory leaving the bounded area toward the end of the battle.

d) This case involving a 1x2 scenario with an unstable equilibrium is similar to case three which was a 1x1 scenario involving one unstable equilibrium.

F. (2X2) CASE COMPARISONS

The next step in the experiment was to develop cases which involved two types of forces on each side and compare the models results for the four modes of behavior discussed. Cases six through nine are 2x2 case scenarios. Their results are shown in Figures E.19 through E.46. The results for each of these cases will be discussed individually.

1. Case Six

Case six is a 2x2 force level scenario designed to test the models when there are no equilibria in the state space. The results for case six are shown in Figures E.19 through E.25. The following observations are noted from the results:

a) The resulting trajectories shown in the Total Force Level Trajectory plot in Figure E.19 show both models behaved similarly. In this particular case the Y force was the winning force. The expected behavior for no equilibria was observed in this case.

b) The M/W model results were within the defined one standard deviation boundaries during most of the battle, leaving the boundaries only toward the end of the engagement.

c) Further comparison of the two models can be done using the results plotted for the individual forces in Figures E.20 through E.25. From these results it is evident that the X force results are the ones where the models differed the most. The trajectories all had the same general trends.

d) This 2x2 case involving no equilibria is similar to case one which was a 1x1 case scenario involving no equilibria.

2. **Case Seven**

Case seven was designed to test the models results with two type X forces and two type Y forces when two equilibria exist. The results of the two models are shown in Figures E.26 through E.32. The following observations are noted for case seven:

a) The resulting trajectories for both models behaved in the expected manner. They approached the unstable equilibrium and then turned away and headed off. In this particular case the X force is winning, but the results would be different if the force levels were such that the initial point was above the separatrix.

b) The M/W trajectory had the same shape as the C/S trajectory. The M/W trajectory shown in Figure E.26 was outside the established boundaries during the initial portion of the battle, but was within the boundaries after they turned away from the equilibrium. However, examination of the individual units force level trajectories shown in Figures E.27 through E.32 shows the M/W model within the boundaries for the X force and just outside the boundaries for the Y force.

c) This 2x2 case involving two equilibria is similar to case four which was a 1x1 scenario involving two equilibria.

3. **Case Eight**

Case eight was designed to test the two models results for a force scenario involving two type X forces and two type Y forces with one stable equilibrium present in the state space. The results for case eight are shown in Figure E.33 through E.39. The following observations are noted for case eight:

a) The resulting trajectories for the M/W and C/S models both exhibited the expected behavior with one stable equilibrium present. They both were attracted to the stable equilibrium point and remained in its vicinity. The C/S model shows more variability due to its stochastic process.

b) The M/W trajectory had the same shape and remained within the established boundaries for the entire battle.

c) This 2x2 case involving one stable equilibrium is similar to case two which was a 1x1 scenario involving one stable equilibrium.

4. **Case Nine**

Case nine was designed to test the two models results for a 2x2 force level scenario when one unstable equilibrium exists. The results for case nine are shown in Figures E.40 through E.46. The following observations are noted for case nine:

a) The resulting trajectories for each of the models behaved in the expected manner for an unstable equilibrium.

b) The M/W trajectory had the same general shape, approaching and turning away from the equilibrium, but it was not within the designated boundaries during most of the battle. Reviewing the results of the individual units shows that the two models did not behave similarly as a function of time.

c) This 2x2 case involving one unstable equilibrium is similar to case three which is a 1x1 scenario involving one unstable equilibrium.

G. (3X3) CASE COMPARISONS

The last force level scenario developed for this experiment involves three type X forces and three type Y forces. One case, labeled as a 3x3 case, was developed for this experiment. There is no discussion of the two models results for each of the four modes of behavior discussed earlier for the 3x3 scenario. Rather, one case was run to test the models results for the larger 3x3 scenario.

1. **Case Ten**

Case ten is the one 3x3 case developed for comparing the M/W model results with the C/S model results. The following observations are made from the results shown in Figures E.47 through E.55:

a) The M/W trajectory was well within the established boundaries throughout the battle and the two trajectories shapes were almost identical.

b) The M/W and C/S trajectories for each of the individual units are very similar. In several of the cases the two trajectories are overlapping for most of the battle.

H. SUMMARY OF RESULTS

The results of the numerical experiment, with ten cases involving 1x1, 1x2, 2x2, and 3x3 force level scenarios, have been presented and briefly discussed in this chapter. Some general observations from these results will serve as a summary of the experiment.

First, the results for the ten cases support the idea that a deterministic attrition model is often a good approximation for the mean result of a stochastic attrition model. From the results shown in the previous section, only one of the ten cases displayed force level trajectories where the M/W model was significantly different from the C/S model. Five of the ten cases showed the M/W trajectories were within the established one standard deviation boundary for the entire battle (cases one, three, four, eight, and ten). Of the remaining cases where the deterministic M/W model results were outside the boundary for a portion of the battle shown in the Total Force Level Trajectory plot, it was noted that the individual unit force level trajectories were within the established one standard deviation boundary in all but one case (case nine).

Secondly, the results from the experiment show that both models force level trajectories displayed the expected mode of behavior for the number and type of equilibrium points which were present in all cases. This shows strong support for the research done by Professors Moose and Wozencraft.

This experiments results are certainly not proof that the deterministic model is always a good approximation for the mean result of a stochastic model. However, the results do indicate they often may be a good approximation.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The results of this research showed that the M/W trajectory was a good approximation for the C/S trajectory in nine of ten cases. This is an indication that deterministic attrition models may be good approximations for the mean of stochastic attrition models. It has also shown that the modes of behavior for both models trajectories are very similar. The combination of these two results provides strong support that the M/W model is a good approximation for the C/S model. These results provide important support for the argument that a deterministic attrition model can be used to model the combat attrition process which is generally agreed upon as a stochastic process.

This simulation provided a test of the Moose/Wozencraft model. The M/W model was tested utilizing four different force composition scenarios, for each of the four types of trajectory behavior modes. The results showed the M/W models results were good approximations for the individual unit level and the aggregate force levels of the C/S model. This Generalized Lanchester Equation type model is simple to use and understand. It allows the modeler, or military planner, to model an aggregate force of as many different type forces as required to model the real-world battlefield. The M/W model has real potential for future use by military analysts.

B. COMMAND AND CONTROL APPLICATIONS

A key decision in the Command and Control decision process is what force structure is required to engage a proposed enemy. The problem is to determine the optimum force size and composition required to engage an enemy and win. Optimizing one's force is a very complex problem, involving the right mixture of force types and the right number of each of the forces. An example would be when a commander may need a tank heavy unit for a specific battle in order to ensure that his force will win.

A model such as the M/W model can serve as a very useful decision aid for the military planners and combat commanders to assist with optimization of forces on the battlefield. Given an accepted set of attrition-rate coefficients, the model can provide the capability of testing for the optimum mix and force size required to fight and win a battle. The use of a model of this type by maneuver units would require the commanders and their staffs to ask the intelligence community for specific information on the enemies current force structure and resupply capability. This information could be used with the model to assist planners and commanders to decide whether to engage, and the optimum force compostion if the decision is made to do so. If not, they may chose to with-draw and fight on better terms. Models of this type would be beneficial to military planners responsible for developing OPLANS and CONPLANS also. It would allow them to hypothesize the expected enemy force structures and test proposed force structures to engage the enemy.

C. RECOMMENDATIONS

After completing this project it is apparent that a great deal of future research can be done with these two models to provide a more complete analysis. The results from this initial survey have identified areas for further testing which would provide a more complete understanding of the dynamics of the two models. The areas listed below would provide good topics for future work with these models:

1) Utilize the models to locate the equilibrium points and the separatrix for cases where an unstable equilibrium point exits in the state space. Once they have been located, observe the two models results in the vicinity of the equilibrium point for several different inital force levels. The separatrix is a very interesting factor which should be observed closely.

2) A detailed analysis of case nine of this experiment is recommended. Case nine was the one case which was run that did not behave similarly to the rest of the cases. At this time it is not known why, and therefore deserves further research.

3) One of the observations made from this experiment involved a difference in the attrition-rate of a force over time for the two models. This could be a result of the normalization factor used for the aimed fire attrition in the M/W model. This factor was used to ensure that an individual force was not allowed to fall below a zero level. This factor may cause the two models to have slightly different attrition-rates over time. This can be tested by removing the factor and changing the models computer program to stop if one force falls to a zero level.

4) Test the model(s) against approved combat simulations.

Further research in this area may lead to an attrition model which is accepted as a useful model for modern combat. This could provide a useful Command and Control decision aid for use by commanders and their staffs to optimize force structures on the battlefield.

APPENDIX A
EXEC FILE FOR C/S PROGRAM

```
&TRACE ON
GLOBAL TXTLIB VFORTLIB NONIMSL IMSLSP CMSLIB
GLOBAL LOADLIB VFLODLIB
FILEDEF 4 TERM
FILEDEF 5 DISK INTAW DATA A
FILEDEF 6 DISK TAW  FT06F001 A
FILEDEF 7 DISK TAW  FT07F001 A
FILEDEF 8 DISK TXTY DATA A
FILEDEF 9 DISK X1Y1 DATA A
FILEDEF 10 DISK X2Y2 DATA A
FILEDEF 11 DISK X3Y3 DATA A
FILEDEF 12 DISK STEP DATA A
FILEDEF 13 DISK STATTF DATA A
FILEDEF 14 DISK STATF1 DATA A
FILEDEF 15 DISK STATF2 DATA A
FILEDEF 16 DISK STATF3 DATA A
LOAD TAW
START MAIN
CP MSG 0731P JOB TAW COMPLETED
```

APPENDIX B
C/S COMPUTER PROGRAM

```
C
C                    PROGRAM LANCHESTER
C
C                        6/18/86
C
C         WRITTEN TO TEST THE MOOSE/WOZENCRAFT THEORY
C
C        NTYPX,Y         NUMBER OF TYPES OF COMBATANTS <INPUT>
C        NXS,YS(I)       INITIAL NUMBER OF EACH TYPE OF COMBATANT
C        NX,Y(I)         NUMBER OF EACH TYPE OF COMBATANT ALIVE
C
C        AAA(I,J)        ATTRITION: X(I) LOSSES TO AREA FIRE
C        BBB(I,J)        ATTRITION: Y(J) SHOOTS X
C        CCC(I,J)        ATTRITION: Y(I) LOSSES TO AREA FIRE
C        DDD(I,J)        ATTRITION: X(I) SHOOTS Y
C
C        NRRR(I)         NUMBER COMBATANT X(I) RESUPPLIED/TIMESTEP
C        NSSS(I)         NUMBER COMBATANT Y(I) RESUPPLIED/TIMESTEP
C
C        X,YPOS(K)       STORED POSITION OF EACH COMBATANT
C        X,YSTAT(K)      STORED STATUS (LIVE OR DEAD) OF EACH COMBATANT
C        NQX,Y(K)        STORED TYPE OF EACH COMBATANT
C
C        PKHX,Y(I,J)     PK: X SHOOTS Y TYPE (I,J) INTERACTION <INPUT>
C        X,YAF(I,J)      AREA FIRE LOSS COEFFICIENTS FOR X <INPUT>
C
C        RN(K)           LISTS OF RANDOM VARIANTS, CONTROLS WHO SHOOTS
C        RX,Y(K)         LISTS OF RANDOM VARIANTS, CONTROLS HITS
C        RXS,YS(K)       LISTS OF RANDOM VARIANTS, CONTROLS SELF-LOSSES
C        RXT,YT(K)       LISTS OF RANDOM VARIANTS, CONTROLS TARGET CHOICE
C
C        PKX,Y(I,J)      PK: X(I) KILLS Y(J) THIS TIMESTEP
C        KAX,Y(I,J)      NR OF Y(J) HIT BY X(I) THIS TIMESTEP, AREA FIRE
C        KHX,Y(I,J)      NR OF Y(J) HIT BY X(I) THIS TIMESTEP, AIMED FIRE
C        PKSX,Y(I)       PROBABILITY OF SELF-LOSS
C
C        NDHX,Y(I)       X,Y(I) LOSSES TO AIMED FIRE THIS TIMESTEP
C        NDAX,Y(I)       X,Y(I) LOSSES TO AREA FIRE THIS TIMESTEP
C        NDSX,Y(I)       X,Y(I) SELF LOSSES THIS TIMESTEP
C
C        NSTOP           MAXIMUM NUMBER OF TIMESTEPS <INPUT>
C        X,YLOC          POSSIBLE POSITIONS OF COMBATANTS <INPUT>
C
      IMPLICIT REAL*8 (D)
C
      CHARACTER*40 IHEAD
      CHARACTER*1  IANS,IYES
      DIMENSION XPOS(3222),XSTAT(3222),NQX(3222),NX(3)
      DIMENSION YPOS(3222),YSTAT(3222),NQY(3222),NY(3)
C
      DIMENSION XLOC(3),PKSX(3),NRRR(3)
      DIMENSION YLOC(3),PKSY(3),NSSS(3)
      DIMENSION XAF(3,3),YAF(3,3)
      DIMENSION PKX(3,3),PKY(3,3)
      DIMENSION PKHX(3,3),PKHY(3,3),KHX(3,3),KHY(3,3)
      DIMENSION PKAX(3,3),PKAY(3,3),KAX(3,3),KAY(3,3)
      DIMENSION NDX(3),NDY(3),NXS(3),NYS(3)
      DIMENSION NDHX(3),NDHY(3)
      DIMENSION NDAX(3),NDAY(3)
      DIMENSION NDSX(3),NDSY(3)
      DIMENSION AAA(3,3),AAAAV(3,3),SAAAAV(3,3)
```

```
      DIMENSION BBB(3,3),BBBAV(3,3),SBBBAV(3,3)
      DIMENSION CCC(3,3),CCCAV(3,3),SCCCAV(3,3)
      DIMENSION DDD(3,3),DDDAV(3,3),SDDDAV(3,3)
C
      DIMENSION RX(4222),RY(4222),RN(9888)
      DIMENSION RXS(4222),RYS(4222)
      DIMENSION RXT(4222),RYT(4222)
C
      DIMENSION SNTX(212),SNX1(212),SNX2(212),SNX3(212)
      DIMENSION SNTY(212),SNY1(212),SNY2(212),SNY3(212)
      DIMENSION SSNTX(212),SSNX1(212),SSNX2(212),SSNX3(212)
      DIMENSION SSNTY(212),SSNY1(212),SSNY2(212),SSNY3(212)
      DIMENSION ANTX(212),ANX1(212),ANX2(212),ANX3(212)
      DIMENSION ANTY(212),ANY1(212),ANY2(212),ANY3(212)
      DIMENSION SDNTX(212),SDNX1(212),SDNX2(212),SDNX3(212)
      DIMENSION SDNTY(212),SDNY1(212),SDNY2(212),SDNY3(212)
      DIMENSION PSNTX(212),PSNX1(212),PSNX2(212),PSNX3(212)
      DIMENSION PSNTY(212),PSNY1(212),PSNY2(212),PSNY3(212)
      DIMENSION TSNTX(212),TSNX1(212),TSNX2(212),TSNX3(212)
      DIMENSION TSNTY(212),TSNY1(212),TSNY2(212),TSNY3(212)
      DIMENSION CSTEP(100)
C
      DATA IYES/'Y'/
C
C
      CALL EXCMS('CLRSCRN')
      READ (5,9400) IHEAD
      WRITE(4,9410)
 9410 FORMAT(' DO YOU WANT TO CHANGE THIS HEADING? (Y/N) (A40)')
      WRITE(4,9400) IHEAD
      READ (4,9405) IANS
 9405 FORMAT(A1)
      IF(IANS.EQ.IYES)  READ (4,9400) IHEAD
 9400 FORMAT(A40)
C
C   ASK USER FOR THE NUMBER OF REPLICATIONS DESIRED
C
      WRITE(4,9490)
 9490 FORMAT('  HOW MANY REPLICAS DO DESIRE? (MAX 100)')
      READ(4,9495) L
 9495 FORMAT(I4)
      WRITE(4,9480)
 9480 FORMAT('   DO YOU WANT FILES 6 AND 7 PRINTED? ')
      READ (4,9401) IANS
 9401 FORMAT(A1)
      WRITE(6,9400) IHEAD
      WRITE(7,9400) IHEAD
      WRITE(8,9400) IHEAD
      WRITE(9,9400) IHEAD
      WRITE(10,9400) IHEAD
      WRITE(11,9400) IHEAD
      WRITE(12,9400) IHEAD
      WRITE(13,9400) IHEAD
      WRITE(14,9400) IHEAD
      WRITE(15,9400) IHEAD
      WRITE(16,9400) IHEAD
      READ (5,9500) NSTOP
      READ (5,9500) NTYPX,NX,XLOC,PKSX,NRRR
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9502)
      WRITE(6,9501) NTYPX,NX,XLOC,PKSX,NRRR
      ENDIF
      READ (5,9505) ((PKHX(I,J),J=1,3),I=1,3)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9521)
      WRITE(6,9520) ((PKHX(I,J),J=1,3),I=1,3)
      ENDIF
      READ (5,9505) ((XAF(I,J),J=1,3),I=1,3)
      IF(IANS.EQ.IYES)THEN
```

```
      WRITE(6,9523)
      WRITE(6,9520) ((XAF(I,J),J=1,3),I=1,3)
      ENDIF
      READ (5,9500) NTYPY,NY,YLOC,PKSY,NSSS
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9503)
      WRITE(6,9501) NTYPY,NY,YLOC,PKSY,NSSS
      ENDIF
      READ (5,9505) ((PKHY(I,J),J=1,3),I=1,3)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9522)
      WRITE(6,9520) ((PKHY(I,J),J=1,3),I=1,3)
      ENDIF
      READ (5,9505) ((YAF(I,J),J=1,3),I=1,3)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9524)
      WRITE(6,9520) ((YAF(I,J),J=1,3),I=1,3)
      ENDIF
 9500 FORMAT(4I5,3F5.1,3F7.3,3I5)
 9501 FORMAT(4I5,3F5.1,3F7.3,3I5)
 9502 FORMAT( ' # X TYPES  NUMBER EACH    LOCATIONS      SELF-LOSS    ',
     X      '   RE-SUPPLY     ')
 9503 FORMAT( ' # Y TYPES  NUMBER EACH    LOCATIONS      SELF-LOSS    ',
     X      '   RE-SUPPLY     ')
 9505 FORMAT(3F8.5)
 9520 FORMAT(9F8.5,/)
 9521 FORMAT(/,' KILL PROBABILITIES FOR FORCE X(I) SHOOTS Y(J)',//,
     X         ' PK(1,1) PK(1,2) PK(1,3) PK(2,1) PK(2,2) PK(2,3)',
     X         ' PK(3,1) PK(3,2) PK(3,3)')
 9522 FORMAT(/,' KILL PROBABILITIES FOR FORCE Y(I) SHOOTS X(J)',//,
     X         ' PK(1,1) PK(1,2) PK(1,3) PK(2,1) PK(2,2) PK(2,3)',
     X         ' PK(3,1) PK(3,2) PK(3,3)')
 9524 FORMAT(/,' AREA FIRE COEFFICIENTS FORCE Y(I) SHOOTS X(J) ',//,
     X         ' AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3)',
     X         ' AA(3,1) AA(3,2) AA(3,3)')
 9523 FORMAT(/,' AREA FIRE COEFFICIENTS FORCE X(I) SHOOTS Y(J) ',//,
     X         ' CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3)',
     X         ' CC(3,1) CC(3,2) CC(3,3)')
C
C           INITIALIZE THE RANDOM NUMBER GENERATORS
C
      READ(5,9510) DX,DY,DN
      DSEEDX=DX
      DSEEDY=DY
      DSEEDN=DN
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9511)
      WRITE(6,9510) DX,DY,DN,DSEEDX,DSEEDY,DSEEDN
      ENDIF
 9510 FORMAT(3D15.8,3I11)
 9511 FORMAT('    DOUBLE PRECISION RANDOM VARIANT SEEDS       ',
     X      '    INTEGER SEED VALUES')
C
C     INITILAIZE THE STATISTICS VARIABLES
C
      DO 10 I=1, 212
         SNTX(I)=0
         SNX1(I)=0
         SNX2(I)=0
         SNX3(I)=0
         SNTY(I)=0
         SNY1(I)=0
         SNY2(I)=0
         SNY3(I)=0
         SSNTX(I)=0
         SSNX1(I)=0
         SSNX2(I)=0
         SSNX3(I)=0
         SSNTY(I)=0
```

```
          SSNY1(I)=0
          SSNY2(I)=0
          SSNY3(I)=0
          ANTX(I)=0
          ANX1(I)=0
          ANX2(I)=0
          ANX3(I)=0
          ANTY(I)=0
          ANY1(I)=0
          ANY2(I)=0
          ANY3(I)=0
          SDNTX(I)=0
          SDNX1(I)=0
          SDNX2(I)=0
          SDNX3(I)=0
          SDNTY(I)=0
          SDNY1(I)=0
          SDNY2(I)=0
          SDNY3(I)=0
          PSNTX(I)=0
          PSNX1(I)=0
          PSNX2(I)=0
          PSNX3(I)=0
          PSNTY(I)=0
          PSNY1(I)=0
          PSNY2(I)=0
          PSNY3(I)=0
          TSNTX(I)=0
          TSNX1(I)=0
          TSNX2(I)=0
          TSNX3(I)=0
          TSNTY(I)=0
          TSNY1(I)=0
          TSNY2(I)=0
          TSNY3(I)=0
 10   CONTINUE
       DO 15 I=1,3
       NYS(I)=NY(I)
       NXS(I)=NX(I)
   15 CONTINUE
       MINST=99
       AVSTEP=0.0
       SDSTEP=0.0
       SSTEP=0.0
       SSSTEP=0.0
       DO 17 I=1,100
          CSTEP(I)=0.0
17    CONTINUE
C
C     BEGIN REPLICATIONS
C
       DO 4000 N=1,L
       NSTEP=0
       NTX=0
       NTY=0
       DO 20 I=1,3
       NY(I)=NYS(I)
       NX(I)=NXS(I)
       IF(I.LE.NTYPX)  THEN
          NTX=NTX+NX(I)
       ENDIF
       IF(I.LE.NTYPY)  THEN
          NTY=NTY+NY(I)
       ENDIF
   20 CONTINUE
       NTXS=NTX
       NTYS=NTY
       NXSTOP=50
       NYSTOP=50
```

```
C
C             INITIALIZE THE TWO FORCES
C
      IMAX=0
      IMIN=1
      IP=0
      DO 200 J=1,NTYPX
      IMAX=IMAX+NX(J)
      DO 100 I=IMIN,IMAX
      IP=IP+1
      XPOS(IP)=XLOC(J)
      XSTAT(IP)=1.0
      NQX(IP)=J
  100 CONTINUE
      IMIN=IMAX+1
  200 CONTINUE
C
      IMAX=0
      IMIN=1
      IP=0
      DO 400 J=1,NTYPY
      IMAX=IMAX+NY(J)
      DO 300 I=IMIN,IMAX
      IP=IP+1
      YPOS(IP)=YLOC(J)
      YSTAT(IP)=1.0
      NQY(IP)=J
  300 CONTINUE
      IMIN=IMAX+1
  400 CONTINUE
      DO 500 I=1,3
      NDX(I)=0
      NDY(I)=0
  500 CONTINUE
      DO 700 I=1,3
      DO 700 J=1,3
      AAAAV(I,J)=0.0
      BBBAV(I,J)=0.0
      CCCAV(I,J)=0.0
      DDDAV(I,J)=0.0
  700 CONTINUE
C
C             SETUP A FILE FOR FUTURE PLOTTING
C
      IF(IANS.EQ.IYES)THEN
      WRITE(7,9700)
 9700 FORMAT(' # NTXS  NTX  NX1  NX2  NX3 DX1 DX2 DX3',
     X       ' NTYS  NTY  NY1  NY2  NY3 DY1 DY2 DY3')
      WRITE(7,9710) NSTEP,NTXS,NTX,NX,NDX,NTYS,NTY,NY,NDY
 9710 FORMAT('+',I4,2(5I5,3I4),/)
      WRITE(6,9711)
 9711 FORMAT(' #  NTXS  NTX  NX1  NX2  NX3 RX1 RX2 RX3',
     X       ' NTYS  NTY  NY1  NY2  NY3 RY1 RY2 RY3')
      WRITE(6,9710) NSTEP,NTXS,NTX,NX,NRRR,NTYS,NTY,NY,NSSS
      END IF
C
C       NTT         TOTAL NUMBER OF LIVE COMBATANTS
C       NSX,Y       NUMBER OF COMBATANT WHO IS FIRING
C       IX,Y        TYPE OF COMBATANT FIRING
C       NTARX,Y     COUNT OF TARGETS ATTACKED
C       NCX,Y       NUMBER OF TARGET UNDER FIRE
C       JX,Y        TYPE OF TARGET UNDER FIRE
C
C             THIS IS THE BEGINNING OF THE COMBAT CYCLE
C
 1000 CONTINUE
      NSTEP=NSTEP+1
      NTT=NTX+NTY
      NGT=1.5*REAL(NTT)
```

```
      NGX=1.3*REAL(NTX)
      NGY=1.3*REAL(NTY)
C
C            MAKE THE LISTS OF RANDOM NUMBERS
C
      CALL GGUBS(DSEEDN,NGT,RN)
      CALL GGUBS(DSEEDX,NGX,RX)
      CALL GGUBS(DSEEDX,NGX,RXS)
      CALL GGUBS(DSEEDX,NGX,RXT)
      CALL GGUBS(DSEEDY,NGY,RY)
      CALL GGUBS(DSEEDY,NGY,RYS)
      CALL GGUBS(DSEEDY,NGY,RYT)
C
C            ZERO STORAGE FOR THIS TIMESTEP
C
      NSX=0
      NTARX=0
      NSY=0
      NTARY=0
      KSTOPX=0
      KSTOPY=0
      DO 1040 I=1,3
      NDX(I)=0
      NDY(I)=0
      NDHX(I)=0
      NDHY(I)=0
      NDAX(I)=0
      NDAY(I)=0
      NDSX(I)=0
      NDSY(I)=0
      DO 1020 J=1,3
      KAX(I,J)=0
      KAY(I,J)=0
      KHX(I,J)=0
      KHY(I,J)=0
 1020 CONTINUE
 1040 CONTINUE
      DO 1060 I=1,3
      DO 1060 J=1,3
      PKAX(I,J)=XAF(I,J)*NTY
      PKAY(I,J)=YAF(I,J)*NTX
 1060 CONTINUE
 1080 CONTINUE
      DO 1090 J=1,3
      DO 1090 I=1,3
      PKX(I,J)=PKAX(I,J)+PKHX(I,J)
      PKY(I,J)=PKAY(I,J)+PKHY(I,J)
 1090 CONTINUE
C
C            DO AIMED AND AREA FIRE INTERACTIONS FOR THIS TIMESTEP
C
C            CHOOSE THE COMBATANT WHO SHOOTS, USING RN(I)
C
C            MAKE COMBATANT NUMBER (COUNT), NSX,Y
C                 COMBATANT MUST BE ALIVE
C                 DETERMINE COMBATANT TYPE, LX,Y
C                 COMBATANT MUST NOT BE A SELF-LOSS
C
C            INCREASE THE TARGET COUNT, NTARX,Y
C                 MAKE NUMBER OF TARGET, NCX,Y, WITH RXT,YT
C                 DETERMINE TARGET TYPE, LX,Y
C
C            COMPARE: PKX,Y(LX,LY) > RX,Y, TARGET IS HIT
C                     PKX,Y(LX,LY) < RX,Y, MISS
C                     PKHX,Y(LX,LY) > RX,Y, HIT BY AIMED FIRE
C                     PKHX,Y(LX,LY) < RX,Y, HIT BY AIMED FIRE
C
 1100 CONTINUE
      DO 1180 M=1,NGT
```

```
          IF(RN(M).LE.(REAL(NTX)/NTT)) THEN
C
C              X SHOOTS
C
              NSX=NSX+1
              IF(NSX.GT.NTX)               GO TO 1120
              IF(XSTAT(NSX).EQ.0.0)        GO TO 1180
              LX=NOX(NSX)
              IF(PKSX(LX).LT.RXS(NSX)) THEN
                  NTARY=NTARY+1
                  NCY=RYT(NTARY)*NTY+1
                  LY=NOY(NCY)
                  IF(PKX(LX,LY).GE.RX(NSX))  THEN
C
C         KILL TARGET NUMBER NCY
C
                      IF(YSTAT(NCY).GT.0.0) THEN
                          YSTAT(NCY)=0.0
                          IF(PKHX(LX,LY).GE.RX(NSX))  THEN
                              NDHY(LY)=NDHY(LY)+1
                              KHX(LX,LY)=KHX(LX,LY)+1
                          ELSE
                              NDAY(LY)=NDAY(LY)+1
                              KAX(LX,LY)=KAX(LX,LY)+1
                          ENDIF
                      ENDIF
                  ENDIF
                  GO TO 1180
              ELSE
                  XSTAT(NSX)=0.0
                  NDSX(LX)=NDSX(LX)+1
                  GO TO 1180
              ENDIF
          ELSE
C
C              Y SHOOTS
C
              NSY=NSY+1
              IF(NSY.GT.NTY)               GO TO 1150
              IF(YSTAT(NSY).EQ.0.0)        GO TO 1180
              LY=NOY(NSY)
              IF(PKSY(LY).LT.RYS(NSY))  THEN
                  NTARX=NTARX+1
                  NCX=RXT(NTARX)*NTX+1
                  LX=NOX(NCX)
                  IF(PKY(LY,LX).GE.RY(NSY))  THEN
C
C           KILL TARGET NUMBER NCX
C
                      IF(XSTAT(NCX).GT.0.0) THEN
                          XSTAT(NCX)=0.0
                          IF(PKHY(LY,LX).GE.RY(NSY))  THEN
                              NDHX(LX)=NDHX(LX)+1
                              KHY(LY,LX)=KHY(LY,LX)+1
                          ELSE
                              NDAX(LX)=NDAX(LX)+1
                              KAY(LY,LX)=KAY(LY,LX)+1
                          ENDIF
                      ENDIF
                  ENDIF
                  GO TO 1180
              ELSE
                  YSTAT(NSY)=0.0
                  NDSY(LY)=NDSY(LY)+1
                  GO TO 1180
              ENDIF
          ENDIF
C
C              ALL TYPE X COMBATANTS HAVE FINISHED FIRING
```

```
C
 1120 CONTINUE
        KSTOPX=1
        GO TO 1170
C
C              ALL TYPE Y COMBATANTS HAVE FINISHED FIRING
C
 1150 CONTINUE
        KSTOPY=1
C
C              EVERY ONE HAS FINISHED FIRING
C
 1170 CONTINUE
        IF(KSTOPX.EQ.1 .AND. KSTOPY.EQ.1)     GO TO 1200
 1180 CONTINUE
C
C              CALCULATE THE KILL RATIOS
C
 1200 CONTINUE
        DO 1290 J=1,NTYPX
        DO 1250 I=1,NTYPY
        NUMD=NX(J)*NY(I)
        IF (NUMD.GE.1) THEN
           DDD(J,I)=REAL(KHX(J,I)*NTY)/REAL(NUMD)
        ELSE
           DDD(J,I)=0.0
        ENDIF
        DDDAV(J,I)=DDDAV(J,I)+DDD(J,I)
        NUMB=NY(I)*NX(J)
        IF (NUMB.GE.1) THEN
           BBB(I,J)=REAL(KHY(I,J)*NTX)/REAL(NUMB)
        ELSE
           BBB(I,J)=0.0
        ENDIF
        BBBAV(I,J)=BBBAV(I,J)+BBB(I,J)
        NUMC=NX(J)*NY(I)
        IF (NUMC.GE.1) THEN
           CCC(J,I)=REAL(KAX(J,I))/REAL(NUMC)
        ELSE
           CCC(J,I)=0.0
        ENDIF
        CCCAV(J,I)=CCCAV(J,I)+CCC(J,I)
        NUMA=NY(I)*NX(J)
        IF (NUMA.GE.1) THEN
           AAA(I,J)=REAL(KAY(I,J))/REAL(NUMA)
        ELSE
           AAA(I,J)=0.0
        ENDIF
        AAAAV(I,J)=AAAAV(I,J)+AAA(I,J)
 1250 CONTINUE
C
C              TOTAL KILLED BY ALL MECHANISMS
C
        NDX(J)=NDHX(J)+NDAX(J)+NDSX(J)
        NDY(J)=NDHY(J)+NDAY(J)+NDSY(J)
 1290 CONTINUE
        IF(IANS.EQ.IYES)THEN
        WRITE(6,9660) NSTEP,((KHX(I,J),J=1,3),I=1,3)
        ENDIF
 9660 FORMAT(/,' TIMESTEP', I4,/,
     X   10X,' AIMED FIRE KILLS OF Y"S BY X FORCE'
     X   ,//,'  D(1,1)  D(1,2)  D(1,3)  D(2,1)  D(2,2)  D(2,3)',
     X      '  D(3,1)  D(3,2)  D(3,3)',/,1X,
     X           9I8,/)
        IF(IANS.EQ.IYES)THEN
        WRITE(6,9665) ((KHY(I,J),J=1,3),I=1,3)
        ENDIF
 9665 FORMAT(10X,' AIMED FIRE KILLS OF X"S BY Y FORCE             ',
     X        //,'  B(1,1)  B(1,2)  B(1,3)  B(2,1)  B(2,2)  B(2,3)',
```

```
     X              '  B(3,1)  B(3,2)  B(3,3)',/,1X,
     X                 9I8,/)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9662) ((KAX(I,J),J=1,3),I=1,3)
      ENDIF
 9662 FORMAT(10X,' AREA FIRE KILLS OF Y"S  BY X FORCE            ',
     X          //,'  C(1,1)  C(1,2)  C(1,3)  C(2,1)  C(2,2)  C(2,3)',
     X              '  C(3,1)  C(3,2)  C(3,3)',/,1X,
     X                 9I8,/)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9668) ((KAY(I,J),J=1,3),I=1,3)
      ENDIF
 9668 FORMAT(10X,' AREA FIRE KILLS OF X"S  BY Y FORCE            ',
     X          //,'  A(1,1)  A(1,2)  A(1,3)  A(2,1)  A(2,2)  A(2,3)',
     X              '  A(3,1)  A(3,2)  A(3,3)',/,1X,
     X                 9I8,/)
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9700)
      WRITE(6,9710) NSTEP,NTXS,NTX,NX,NDX,NTYS,NTY,NY,NDY
      WRITE(7,9651)
      ENDIF
 9651 FORMAT('  D,B,C,A BY ROWS IN DESCENDING ORDER ')
      IF(IANS.EQ.IYES)THEN
      WRITE(7,9650) NSTEP,((KHX(I,J),J=1,3),I=1,3)
      WRITE(7,9650) NSTEP,((KHY(I,J),J=1,3),I=1,3)
      WRITE(7,9650) NSTEP,((KAX(I,J),J=1,3),I=1,3)
      WRITE(7,9650) NSTEP,((KAY(I,J),J=1,3),I=1,3)
      ENDIF
 9650 FORMAT(I4,9I8)
C
C             CALCULATE TOTAL LOSSES FOR ALL INTERACTIONS
C
 1500 CONTINUE
      NDTX=0
      NDTY=0
      DO 1510 J=1,3
      NDTX=NDTX+NDX(J)
      NDTY=NDTY+NDY(J)
 1510 CONTINUE
      IF(IANS.EQ.IYES)THEN
      WRITE(6,9670) NDTX,NRRR,NDHX,NDAX,NDSX,
     X              NDTY,NSSS,NDHY,NDAY,NDSY
      WRITE(7,9670) NDTX,NRRR,NDHX,NDAX,NDSX,
     X              NDTY,NSSS,NDHY,NDAY,NDSY
 9670 FORMAT(' TOTAL X FORCE LOSS WAS',I4,'  RESUPPLY WAS:',3I4,/,
     X       '     AIMED FIRE LOSSES:',3I4,/,
     X       '      AREA FIRE LOSSES:',3I4,/,
     X       '           SELF LOSSES:',3I4,/,
     X       ' TOTAL Y FORCE LOSS WAS',I4,'  RESUPPLY WAS:',3I4,/,
     X       '     AIMED FIRE LOSSES:',3I4,/,
     X       '      AREA FIRE LOSSES:',3I4,/,
     X       '           SELF LOSSES:',3I4)
      ENDIF
C
C             ADD IN RESUPPLIED UNITS
C
 1600 CONTINUE
      DO 1650 K=1,3
      IF(K.LE.NTYPX .AND. NRRR(K).NE.0)  THEN
          NADDX=NRRR(K)
          NX(K)=NX(K)+NADDX
          DO 1620 I=1,NADDX
          NTX=NTX+1
          XPOS(NTX)=XLOC(K)
          NOX(NTX)=K
          XSTAT(NTX)=1.0
 1620   CONTINUE
      ENDIF
      IF(K.LE.NTYPY .AND. NSSS(K).NE.0)  THEN
```

```
          NADDY=NSSS(K)
          NY(K)=NY(K)+NADDY
          DO 1640 I=1,NADDY
          NTY=NTY+1
          YPOS(NTY)=YLOC(K)
          NOY(NTY)=K
          YSTAT(NTY)=1.0
 1640     CONTINUE
        ENDIF
 1650 CONTINUE
C
C             CLOSE OUT DESTROYED UNITS
C
 1700 CONTINUE
      DO 1710 J=1,3
      NX(J)=0
      NY(J)=0
 1710 CONTINUE
      IX=0
      DO 1740 I=1,NTX
      IF(XSTAT(I).NE.0.0) THEN
          IX=IX+1
          NOX(IX)=NOX(I)
          XPOS(IX)=XPOS(I)
          XSTAT(IX)=XSTAT(I)
          K=NOX(I)
          NX(K)=NX(K)+1
      ENDIF
 1740 CONTINUE
      NTX=IX
      IY=0
      DO 1780 I=1,NTY
      IF(YSTAT(I).NE.0.0) THEN
          IY=IY+1
          NOY(IY)=NOY(I)
          YPOS(IY)=YPOS(I)
          YSTAT(IY)=YSTAT(I)
          K=NOY(I)
          NY(K)=NY(K)+1
      ENDIF
 1780 CONTINUE
      NTY=IY
C
C       PRINT STATUS FOR EACH TIME STEP IF REQUESTED
C
      IF(IANS.EQ.IYES)THEN
      WRITE(7,9710) NSTEP,NTXS,NTX,NX,NDX,NTYS,NTY,NY,NDY
      WRITE(6,9710) NSTEP,NTXS,NTX,NX,NRRR,NTYS,NTY,NY,NSSS
      END IF
C
C   ACCUMILATE STATISTICAL VARIABLES FOR EACH TIME STEP
C
      I=NSTEP
      SNTX(I)=SNTX(I)+NTX
      SNX1(I)=SNX1(I)+NX(1)
      SNX2(I)=SNX2(I)+NX(2)
      SNX3(I)=SNX3(I)+NX(3)
      SNTY(I)=SNTY(I)+NTY
      SNY1(I)=SNY1(I)+NY(1)
      SNY2(I)=SNY2(I)+NY(2)
      SNY3(I)=SNY3(I)+NY(3)
      SSNTX(I)=SSNTX(I)+NTX**2
      SSNX1(I)=SSNX1(I)+NX(1)**2
      SSNX2(I)=SSNX2(I)+NX(2)**2
      SSNX3(I)=SSNX3(I)+NX(3)**2
      SSNTY(I)=SSNTY(I)+NTY**2
      SSNY1(I)=SSNY1(I)+NY(1)**2
      SSNY2(I)=SSNY2(I)+NY(2)**2
      SSNY3(I)=SSNY3(I)+NY(3)**2
```

```
C
C      TERMINATE IF FORCES ARE ATTRITTED BELOW SPECIFIED LEVEL
C
       IF(NTX.LE.NXSTOP .OR. NTY.LE.NYSTOP)  GO TO 9120
C
C             TERMINATE ON NUMBER OF CYCLES
C
       IF(NSTEP.LT.NSTOP)         GO TO 1000
 9120  CONTINUE
       STEPS=REAL(NSTEP)
C
C  CALCULATE THE AVERAGE ATTRITION COEFFICIENTS PER REPLICATION
C
       DO 9100 J=1,3
       DO 9100 I=1,3
       AAAAV(I,J)=AAAAV(I,J)/STEPS
       BBBAV(I,J)=BBBAV(I,J)/STEPS
       CCCAV(I,J)=CCCAV(I,J)/STEPS
       DDDAV(I,J)=DDDAV(I,J)/STEPS
       SAAAAV(I,J)=SAAAAV(I,J)+AAAAV(I,J)
       SBBBAV(I,J)=SBBBAV(I,J)+BBBAV(I,J)
       SCCCAV(I,J)=SCCCAV(I,J)+CCCAV(I,J)
       SDDDAV(I,J)=SDDDAV(I,J)+DDDAV(I,J)
 9100  CONTINUE
       CSTEP(N)=NSTEP
C
C   ACCUMILATE THE STATISTICAL VARIABLES FOR THE NUMBER OF TIME STEPS
C
       SSTEP=SSTEP+CSTEP(N)
       SSSTEP=SSSTEP+(CSTEP(N)**2)
C
C   WRITE THE NUMBER OF REPLICATIONS AND THE NUMBER OF STEPS TO FILE
C
       WRITE(12,8510)N,CSTEP(N)
 8510  FORMAT(I3,F4.0)
C
C      KEEP TRACK OF MINIMUM NUMBER OF STEPS FOR ALL REPLICATIONS
C
       IF(MINST.GT.CSTEP(N))THEN
         MINST=CSTEP(N)
       ENDIF
 4000  CONTINUE
C
C  CALCULATE AVERAGE/STANDARD DEV.OF FORCE LEVELS FOR EACH TIME STEP
C
       B=REAL(L)
       RD=1.0/B
       RDD=1.0/(B*(B-1))
       DO 4400 I=1,MINST
       ANTX(I)=SNTX(I)*RD
       ANX1(I)=SNX1(I)*RD
       ANX2(I)=SNX2(I)*RD
       ANX3(I)=SNX3(I)*RD
       ANTY(I)=SNTY(I)*RD
       ANY1(I)=SNY1(I)*RD
       ANY2(I)=SNY2(I)*RD
       ANY3(I)=SNY3(I)*RD
       SDNTX(I)=SQRT(ABS((B*SSNTX(I)-SNTX(I)**2)*RDD))
       SDNX1(I)=SQRT(ABS((B*SSNX1(I)-SNX1(I)**2)*RDD))
       SDNX2(I)=SQRT(ABS((B*SSNX2(I)-SNX2(I)**2)*RDD))
       SDNX3(I)=SQRT(ABS((B*SSNX3(I)-SNX3(I)**2)*RDD))
       SDNTY(I)=SQRT(ABS((B*SSNTY(I)-SNTY(I)**2)*RDD))
       SDNY1(I)=SQRT(ABS((B*SSNY1(I)-SNY1(I)**2)*RDD))
       SDNY2(I)=SQRT(ABS((B*SSNY2(I)-SNY2(I)**2)*RDD))
       SDNY3(I)=SQRT(ABS((B*SSNY3(I)-SNY3(I)**2)*RDD))
C
C   CALCULATE THE VALUES FOR PLUS AND MINUS ONE STANDARD DEVIATION
C   FROM THE MEAN VALUES FOR PLOTTING
C
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```
      PSNTX(I)=ANTX(I)+SDNTX(I)
      PSNX1(I)=ANX1(I)+SDNX1(I)
      PSNX2(I)=ANX2(I)+SDNX2(I)
      PSNX3(I)=ANX3(I)+SDNX3(I)
      PSNTY(I)=ANTY(I)+SDNTY(I)
      PSNY1(I)=ANY1(I)+SDNY1(I)
      PSNY2(I)=ANY2(I)+SDNY2(I)
      PSNY3(I)=ANY3(I)+SDNY3(I)
      TSNTX(I)=ANTX(I)-(SDNTX(I))
      TSNX1(I)=ANX1(I)-SDNX1(I)
      TSNX2(I)=ANX2(I)-SDNX2(I)
      TSNX3(I)=ANX3(I)-SDNX3(I)
      TSNTY(I)=ANTY(I)-SDNTY(I)
      TSNY1(I)=ANY1(I)-SDNY1(I)
      TSNY2(I)=ANY2(I)-SDNY2(I)
      TSNY3(I)=ANY3(I)-SDNY3(I)
C
C   CALCULATE THE AVERAGE ATTRITION COEFFICIENTS OVER ALL REPLICATIONS
C
      DO 9200 J=1,3
      DO 9200 K=1,3
        AAAAV(K,J)=SAAAAV(K,J)*RD
        BBBAV(K,J)=SBBBAV(K,J)*RD
        CCCAV(K,J)=SCCCAV(K,J)*RD
        DDDAV(K,J)=SDDDAV(K,J)*RD
9200  CONTINUE
C
C     PRINT STATISTICS TO FILES 8,9,10,11 FOR PLOTTING
C
      WRITE(8,8100) I,ANTX(I),ANTY(I),PSNTX(I),PSNTY(I),TSNTX(I),
     XTSNTY(I)
      WRITE(9,8100) I,ANX1(I),ANY1(I),PSNX1(I),PSNY1(I),TSNX1(I),
     XTSNY1(I)
      WRITE(10,8100) I,ANX2(I),ANY2(I),PSNX2(I),PSNY2(I),TSNX2(I),
     XTSNY2(I)
      WRITE(11,8100) I,ANX3(I),ANY3(I),PSNX3(I),PSNY3(I),TSNX3(I),
     XTSNY3(I)
 8100 FORMAT(I3,6(F8.2,1X))
C
C   WRITE AVG'S AND STANDARD DEVIATIONS TO FILE FOR TABLES
C
      WRITE(13,8150)I,ANTX(I),SDNTX(I),ANTY(I),SDNTY(I)
      WRITE(14,8150)I,ANX1(I),SDNX1(I),ANY1(I),SDNY1(I)
      WRITE(15,8150)I,ANX2(I),SDNX2(I),ANY2(I),SDNY2(I)
      WRITE(16,8150)I,ANX3(I),SDNX3(I),ANY3(I),SDNY3(I)
 8150 FORMAT(I3,4(F8.2,1X))
 4400 CONTINUE
C
C   CALCULTE THE AVG AND STANDARD DEVIATION FOR THE # OF NSTEPS PER RUN
C
      AVSTEP=SSTEP*RD
      SDSTEP=SQRT(ABS((B*SSSTEP-SSTEP**2)*RDD))
      WRITE(12,8500)MINST,AVSTEP,SDSTEP
 8500 FORMAT(I3,2F7.2)
C
C            EXPLAIN CAUSE OF TERMINATION
C
 9000 CONTINUE
      WRITE(7,9656)
 9656 FORMAT('  A,B,C,D BY ROWS IN DESCENDING ORDER ')
      WRITE(7,9655) ((AAAAV(I,J),J=1,3),I=1,3)
      WRITE(7,9655) ((BBBAV(I,J),J=1,3),I=1,3)
 9655 FORMAT(' AV ',9F8.5)
      WRITE(6,9695) ((AAAAV(I,J),J=1,3),I=1,3)
      WRITE(6,9697) ((BBBAV(I,J),J=1,3),I=1,3)
 9690 FORMAT(/,' TIMESTEP', I4,/,
     X 10X,' AVERAGE ATTRITION COEFFICIENTS FOR X FORCE  ( X E+03)',//,
     X       ' CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3)',
     X       ' CC(3,1) CC(3,2) CC(3,3)',/,
```

```
     X          3P,9F8.5,/)
        WRITE(7,9655) ((CCCAV(I,J),J=1,3),I=1,3)
        WRITE(7,9655) ((DDDAV(I,J),J=1,3),I=1,3)
        WRITE(6,9698) ((CCCAV(I,J),J=1,3),I=1,3)
        WRITE(6,9699) ((DDDAV(I,J),J=1,3),I=1,3)
 9695 FORMAT(10X,
     X     ' AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE            ',//,
     X        ' AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3)',
     X        ' AA(3,1) AA(3,2) AA(3,3)',/,1X,
     X          9F8.5,/)
 9697 FORMAT(10X,
     X     ' AVERAGE ATTRITION COEFFICIENTS FOR Y FORCE            ',//,
     X        ' BB(1,1) BB(1,2) BB(1,3) BB(2,1) BB(2,2) BB(2,3)',
     X        ' BB(3,1) BB(3,2) BB(3,3)',/,1X,
     X          9F8.5,/)
 9698 FORMAT(10X,
     X     ' AVERAGE ATTRITION COEFFICIENTS FOR X FORCE            ',//,
     X        ' CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3)',
     X        ' CC(3,1) CC(3,2) CC(3,3)',/,1X,
     X          9F8.5,/)
 9699 FORMAT(10X,
     X     ' AVERAGE ATTRITION COEFFICIENTS FOR X FORCE            ',//,
     X        ' DD(1,1) DD(1,2) DD(1,3) DD(2,1) DD(2,2) DD(2,3)',
     X        ' DD(3,1) DD(3,2) DD(3,3)',/,1X,
     X          9F8.5,/)
C
        IF(NTX.LT.NXSTOP)  THEN
            WRITE (6,9620) NSTEP,NTX,NTXS
            WRITE (7,9620) NSTEP,NTX,NTXS
        ENDIF
        IF(NTY.LT.NYSTOP)  THEN
            WRITE (6,9630) NSTEP,NTY,NTYS
            WRITE (7,9630) NSTEP,NTY,NTYS
        ENDIF
        IF (NSTEP .EQ. NSTOP)  THEN
            WRITE (6,9640) NSTOP
            WRITE (7,9640) NSTOP
        ENDIF
 9620 FORMAT(' STOPPED IN STEP ',I5,' BY FORCE X',/,
     X        ' CURRENT X STRENGTH =',I5,', INITIAL X STRENGTH',I5,//)
 9630 FORMAT(' STOPPED IN STEP ',I5,' BY FORCE Y',/,
     X        ' CURRENT Y STRENGTH =',I5,', INITIAL Y STRENGTH',I5,//)
 9640 FORMAT(' STOPPED BY PROGRAM AT NSTOP = ',I5,//)
        END
```

APPENDIX C
EXEC FILE FOR M/W PROGRAM

```
&TRACE ON
GLOBAL TXTLIB VFORTLIB NONIMSL IMSLSP CMSLIB
GLOBAL LOADLIB VFLODLIB
FILEDEF 4 TERM
FILEDEF 5 DISK INMWM DATA A
FILEDEF 6 DISK MWM  STLIST A
FILEDEF 8 DISK TOTAL DATA A
FILEDEF 9 DISK FOR1 DATA A
FILEDEF 10 DISK FOR2 DATA A
FILEDEF 11 DISK FOR3 DATA A
LOAD MWM
START MAIN
CP MSG 0731P JOB MWM COMPLETED
```

APPENDIX D
M/W COMPUTER PROGRAM

```
C
C     PROGRAM WRITTEN TO RUN THE GENERALIZED LANCHESTER
C
C          EQUATION MODEL DESIGNED BY
C
C     PAUL H. MOOSE AND JACK M. WOZENCRAFT
C
C                   1/08/87
C
C      WRITTEN TO TEST THE MOOSE/WOZENCRAFT THEORY
C
C
C     NTYPX          NUMBER OF TYPES OF X COMBATANTS <INPUT>
C     NTYPY          NUMBER OF TYPES OF Y COMBATANTS <INPUT>
C     TNX(I)         NUMBER OF EACH TYPE OF X COMBATANTS
C     TNY(J)         NUMBER OF EACH TYPE OF Y COMBATANTS
C     STNX,Y         VARIBLE TO SUM TOTAL FORCE LEVEL PER TIMESTEP
C     TTNX,Y         TOTAL FORCE LEVEL PER TIMESTEP
C
C     AAA(J,I)      ATTRITION: X(I) AREA FIRE LOSSES DUE TO Y(J)
C     BBB(J,I)       ATTRITION: X(I) AIMED FIRE LOSSES DUE TO Y(J)
C     CCC(I,J)       ATTRITION: Y(J) ARERA FIRE LOSSES DUE TO X(I)
C     DDD(I,J)       ATTRITION: Y(J) AIMED FIRE LOSSES DUE TO X(I)
C
C     SAAA(I)        ATTRITION: X(I) SUM OF THE AREA LOSSES
C     SBBB(I)        ATTRITION: X(I) SUM OF THE AIMED FIRE LOSSES
C     SCCC(J)        ATTRITION: Y(J) SUM OF THE AREA FIRE LOSSES
C     SDDD(J)        ATTRITION: Y(J) SUM OF THE AIMED FIRE LOSSES
C
C     NRRR(I)        NUMBER COMBATANT X(I) RESUPPLIED/TIMESTEP
C     NSSS(J)        NUMBER COMBATANT Y(J) RESUPPLIED/TIMESTEP
C
C     UUU(I)         SELF-ATTRITION COEFFICIENT FOR X FORCE
C     VVV(J)         SELF-ATTRITION COEFFICIENT FOR Y FORCE
C
C     AMLX(I)        X(I) LOSSES TO AIMED FIRE THIS TIMESTEP
C     AMLY(J)        Y(J) LOSSES TO AIMED FIRE THIS TIMESTEP
C
C     ARLX(I)        X(I) LOSSES TO AREA FIRE THIS TIMESTEP
C     ARLY(J)        Y(J) LOSSES TO AREA FIRE THIS TIMESTEP
C
C     SLX(I)         X(I) SELF LOSSES THIS TIMESTEP
C     SLY(J)         Y(J) SELF LOSSES THIS TIMESTEP
C
C     TNDX(I)        TOTAL CHANGE IN X(I) FORCE LEVEL THIS TIME STEP
C     TNDY(J)        TOTAL CHANGE IN Y(J) FORCE LEVEL THIS TIME STEP
C
C     NSTOP          MAXIMUM NUMBER OF TIMESTEPS <INPUT>
C     XSTOP          X FORCE LEVEL DEFINED AS THE BREAK PT.
C     YSTOP          Y FORCE LEVEL DEFINED AS THE BREAK PT.
C
C
      IMPLICIT REAL*8 (D)
C
      CHARACTER*40 IHEAD
      CHARACTER*1  IANS,IYES
C
      DIMENSION TNX(3),TNY(3)
      DIMENSION NRRR(3),NSSS(3)
      DIMENSION AMLX(3),AMLY(3)
      DIMENSION ARLX(3),ARLY(3)
```

```
      DIMENSION SLX(3),SLY(3)
      DIMENSION TNDX(3),TNDY(3)
      DIMENSION AAA(3,3),SAAA(3)
      DIMENSION BBB(3,3),SBBB(3)
      DIMENSION CCC(3,3),SCCC(3)
      DIMENSION DDD(3,3),SDDD(3)
      DIMENSION UUU(3),VVV(3)
C
C
      DATA IYES/'Y'/
C
C
      CALL EXCMS('CLRSCRN')
      READ (5,9400) IHEAD
      WRITE(4,9410)
 9410 FORMAT(' DO YOU WANT TO CHANGE THIS HEADING? (Y/N) (A40)')
      WRITE(4,9400) IHEAD
      READ (4,9405) IANS
 9405 FORMAT(A1)
      IF(IANS.EQ.IYES)  READ (4,9400) IHEAD
 9400 FORMAT(A40)
 9401 FORMAT(A1)
      WRITE(6,9400) IHEAD
      WRITE(8,9400) IHEAD
      WRITE(9,9400) IHEAD
      WRITE(10,9400) IHEAD
      WRITE(11,9400) IHEAD
      READ (5,9500) NSTOP
      READ (5,9500) NTYPX,TNX,UUU,NRRR
      WRITE(6,9502)
      WRITE(6,9501) NTYPX,TNX,UUU,NRRR
      READ (5,9505) ((DDD(I,J),J=1,3),I=1,3)
      WRITE(6,9521)
      WRITE(6,9520) ((DDD(I,J),J=1,3),I=1,3)
      READ (5,9505) ((CCC(I,J),J=1,3),I=1,3)
      WRITE(6,9523)
      WRITE(6,9520) ((CCC(I,J),J=1,3),I=1,3)
      READ (5,9500) NTYPY,TNY,VVV,NSSS
      WRITE(6,9503)
      WRITE(6,9501) NTYPY,TNY,VVV,NSSS
      READ (5,9505) ((BBB(I,J),J=1,3),I=1,3)
      WRITE(6,9522)
      WRITE(6,9520) ((BBB(I,J),J=1,3),I=1,3)
      READ (5,9505) ((AAA(I,J),J=1,3),I=1,3)
      WRITE(6,9524)
      WRITE(6,9520) ((AAA(I,J),J=1,3),I=1,3)
 9500 FORMAT(I5,3F5.0,3F7.3,3I5)
 9501 FORMAT(I5,3F7.2,3F7.3,3I5)
 9502 FORMAT( ' # X TYPES  NUMBER EACH  SELF-LOSS    ',
     X       '    RE-SUPPLY    ')
 9503 FORMAT( ' # Y TYPES  NUMBER EACH  SELF-LOSS    ',
     X       '    RE-SUPPLY    ')
 9505 FORMAT(3F8.5)
 9520 FORMAT(9F8.5,/)
 9521 FORMAT(/,' KILL PROBABILITIES FOR FORCE X(I) SHOOTS Y(J)',//,
     X         ' DD(1,1) DD(1,2) DD(1,3) DD(2,1) DD(2,2) DD(2,3)',
     X         ' DD(3,1) DD(3,2) DD(3,3)')
 9522 FORMAT(/,' KILL PROBABILITIES FOR FORCE Y(I) SHOOTS X(J)',//,
     X         ' BB(1,1) BB(1,2) BB(1,3) BB(2,1) BB(2,2) BB(2,3)',
     X         ' BB(3,1) BB(3,2) BB(3,3)')
 9524 FORMAT(/,' AREA FIRE COEFFICIENTS FORCE Y(I) SHOOTS X(J) ',//,
     X         ' AA(1,1) AA(1,2) AA(1,3) AA(2,1) AA(2,2) AA(2,3)',
     X         ' AA(3,1) AA(3,2) AA(3,3)')
 9523 FORMAT(/,' AREA FIRE COEFFICIENTS FORCE X(I) SHOOTS Y(J) ',//,
     X         ' CC(1,1) CC(1,2) CC(1,3) CC(2,1) CC(2,2) CC(2,3)',
     X         ' CC(3,1) CC(3,2) CC(3,3)')
C
C           INITIALIZE THE INTIAL FORCES LEVELS
C
```

```
      INTX=0
      INTY=0
      DO 1100 I=1,3
         INTX=INTX+TNX(I)
         INTY=INTY+TNY(I)
 1100 CONTINUE
      TTNX=INTX
      TTNY=INTY
      XSTOP=50.0
      YSTOP=50.0
      MINST=99
C
C              SETUP A FILE FOR TRACING EACH STEPS RESULTS
C
      WRITE(6,9711)
 9711 FORMAT(' #    INTX TTNX TNX1 TNX2 TNX3 RX1 RX2 RX3',
     X       ' INTY TTNY TNY1 TNY2 TNY3 RY1 RY2 RY3')
      WRITE(6,9712) NSTEP,INTX,TTNX,TNX,NRRR,INTY,TTNY,TNY,NSSS
 9712 FORMAT(I4,2(I3,4F7.2,3I3),/)
      WRITE(6,9700)
 9700 FORMAT('   # INTX TTNX TNX1 TNX2 TNX3',
     X       ' INTY TTNY TNY1 TNY2 TNY3')
      WRITE(6,9710) NSTEP,INTX,TTNX,TNX,INTY,TTNY,TNY
 9710 FORMAT(I4,2(I5,4F7.2),/)
C     END IF
C
C
C              THIS IS THE BEGINNING OF THE COMBAT CYCLE
C
      DO 4000 N=1,NSTOP
       NSTEP=N
C
C              ZERO STORAGE FOR THIS TIMESTEP
C
       STNX=0.0
       STNY=0.0
       DO 1015 I=1,NTYPX
        SAAA(I)=0.0
        SBBB(I)=0.0
 1015  CONTINUE
       DO 1020 J=1,NTYPY
        SCCC(J)=0.0
        SDDD(J)=0.0
 1020  CONTINUE
C
C      CALCULATE THE LOSSES DUE TO AREA FIRE FOR THIS TIMESTEP
C
       DO 1110 I=1,NTYPX
       DO 1105 J=1,NTYPY
          SAAA(I)=SAAA(I)+(AAA(J,I)*TNY(J))
          SCCC(J)=SCCC(J)+(CCC(I,J)*TNX(I))
 1105  CONTINUE
 1110  CONTINUE
       DO 1115 I=1,NTYPX
          ARLX(I)=(TNX(I)*SAAA(I))
 1115  CONTINUE
       DO 1120 J=1,NTYPY
          ARLY(J)=TNY(J)*SCCC(J)
 1120  CONTINUE
C
C   CALCULATE THE LOSSES DUE TO AIMED FIRE PER TIMESTEP
C
       DO 1130 I=1,NTYPX
       DO 1125 J=1,NTYPY
          SBBB(I)=SBBB(I)+(BBB(J,I)*TNY(J))
          SDDD(J)=SDDD(J)+(DDD(I,J)*TNX(I))
 1125  CONTINUE
 1130  CONTINUE
       DO 1135 I=1,NTYPX
```

```
          AMLX(I)=(TNX(I)/TTNX)*SBBB(I)
 1135   CONTINUE
          DO 1140 J=1,NTYPY
            AMLY(J)=(TNY(J)/TTNY)*SDDD(J)
 1140   CONTINUE
C
C     CALCULATE THE SELF LOSSES PER TIMESTEP
C
          DO 1142 I=1,NTYPX
            SLX(I)=TNX(I)*UUU(I)
 1142   CONTINUE
          DO 1145 J=1,NTYPY
            SLY(J)=TNY(J)*VVV(J)
 1145   CONTINUE
C
C     CALCULATE THE TOTAL KILLED BY ALL MECHANISMS
C
          DO 1150 I=1,NTYPX
            TNDX(I)=-SLX(I)-ARLX(I)-AMLX(I)+NRRR(I)
 1150   CONTINUE
          DO 1155 J=1,NTYPY
            TNDY(J)=-SLY(J)-ARLY(J)-AMLY(J)+NSSS(J)
 1155   CONTINUE
C
C    CALCULATE THE FORCE LEVELS AFTER THIS TIMESTEP
C
          DO 1160 I=1,NTYPX
            TNX(I)=TNX(I)+TNDX(I)
 1160   CONTINUE
          DO 1165 J=1,NTYPY
            TNY(J)=TNY(J)+TNDY(J)
 1165   CONTINUE
C
C    CALCUTE THE TOTAL X,Y FORCE LEVELS
C
          DO 1170 I=1,NTYPX
            STNX=STNX+TNX(I)
 1170   CONTINUE
          TTNX=STNX
          DO 1175 J=1,NTYPY
            STNY=STNY+TNY(J)
 1175   CONTINUE
          TTNY=STNY
C
C
C         PRINT STATUS FOR EACH TIME STEP
C
        WRITE(6,6710) NSTEP,INTX,TTNX,TNX,INTY,TTNY,TNY
 6710 FORMAT(I4,2(I5,4F7.2),/)
C
C
C
C      PRINT VALUES TO FILES 8,9,10,11 FOR PLOTTING
C
C
C
        WRITE(8,8100) N,TTNX,TTNY
        WRITE(9,8100) N,TNX(1),TNY(1)
        WRITE(10,8100) N,TNX(2),TNY(2)
        WRITE(11,8100) N,TNX(3),TNY(3)
 8100 FORMAT(I3,2(F8.2,1X))
C
C       TERMINATE IF FORCES ARE ATTRITTED BELOW SPECIFIED LEVEL
C
        IF(TTNX.LE.XSTOP .OR.TTNY.LE.YSTOP)  GO TO 5000
C
C    RETURN AND CALULATE FORCE LEVELS FOR ANOTHER TIMESTEP
C
 4000 CONTINUE
```

```
 5000 CONTINUE
C
C             EXPLAIN CAUSE OF TERMINATION
C
C
      IF(TTNX.LT.XSTOP)  THEN
         WRITE (6,9620) NSTEP,TTNX,INTX
      ENDIF
      IF(TTNY.LT.YSTOP)  THEN
         WRITE (6,9630) NSTEP,TTNY,INTY
      ENDIF
      IF (NSTEP .EQ. NSTOP)  THEN
         WRITE (6,9640) NSTOP
      ENDIF
 9620 FORMAT(' STOPPED IN STEP ',I5,' BY FORCE X',/,
     X        ' CURRENT X STRENGTH =',F5.2,', INITIAL X STRENGTH',I5,//)
 9630 FORMAT(' STOPPED IN STEP ',I5,' BY FORCE Y',/,
     X        ' CURRENT Y STRENGTH =',F5.2,', INITIAL Y STRENGTH',I5,//)
 9640 FORMAT(' STOPPED BY PROGRAM AT NSTOP = ',I5,//)
      END
```

APPENDIX E
FORCE LEVEL TRAJECTORIES

This Appendix is designed to provide the reader with a complete set of all of the results of this experiment. The input data sets for each model are presented in tables and then each of the force level trajectories is shown graphically.

TABLE 1

SAMPLE INPUT DATA SET FOR C S MODEL

01 16 87 TEST OF (2X2), 1 UNSTABLE ROOT S
NSTEP
NTYPX NTX1 NTX2 NTX3 loc1 loc2 loc3 u_1 u_2 u_3 r_1 r_2 r_3
d_{11} d_{12} d_{13}
d_{21} d_{22} d_{23}
d_{31} d_{32} d_{33}
c_{11} c_{12} c_{13}
c_{21} c_{22} c_{23}
c_{31} c_{32} c_{33}
NTYPY NTY1 NTY2 NTY3 loc1 loc2 loc3 v_1 v_2 v_3 s_1 s_2 s_3
b_{11} b_{12} b_{13}
b_{21} b_{22} b_{23}
b_{31} b_{32} b_{33}
a_{11} a_{12} a_{13}
a_{21} a_{22} a_{23}
a_{31} a_{32} a_{33}
DSEED DSEED DSEED

TABLE 2

SAMPLE INPUT DATA SET FOR M W MODEL

01 16 87 TEST OF (2X2), 1 UNSTABLE ROOT S
NSTEP
NTYPX NTX1 NTX2 NTX3 u_1 u_2 u_3 r_1 r_2 r_3
d_{11} d_{12} d_{13}
d_{21} d_{22} d_{23}
d_{31} d_{32} d_{33}
c_{11} c_{12} c_{13}
c_{21} c_{22} c_{23}
c_{31} c_{32} c_{33}
NTYPY NTY1 NTY2 NTY3 v_1 v_2 v_3 s_1 s_2 s_3
b_{11} b_{12} b_{13}
b_{21} b_{22} b_{23}
b_{31} b_{32} b_{33}
a_{11} a_{12} a_{13}
a_{21} a_{22} a_{23}
a_{31} a_{32} a_{33}

TABLE 3

INPUT DATA SET FOR C S MODEL CASE 1

```
01 05 87  TEST OF (1X1), NO ROOTS
    99
     1  600    0    0  1.0  1.0  1.0  0.040  0.000  0.000   28    0    0
0.06000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
     1  400    0    0  1.0  1.0  1.0  0.010  0.000  0.000   16    0    0
0.01000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
    345215789.D0    45635761.D0    89342761.D0
```

TABLE 4

INPUT DATA SET FOR M W MODEL CASE 1

```
01 05 87  TEST OF (1X1), NO ROOTS
    99
     1  600    0    0  1.0  1.0  1.0  0.040  0.000  0.000   28    0    0
0.06000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
     1  400    0    0  1.0  1.0  1.0  0.010  0.000  0.000   16    0    0
0.01000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
```

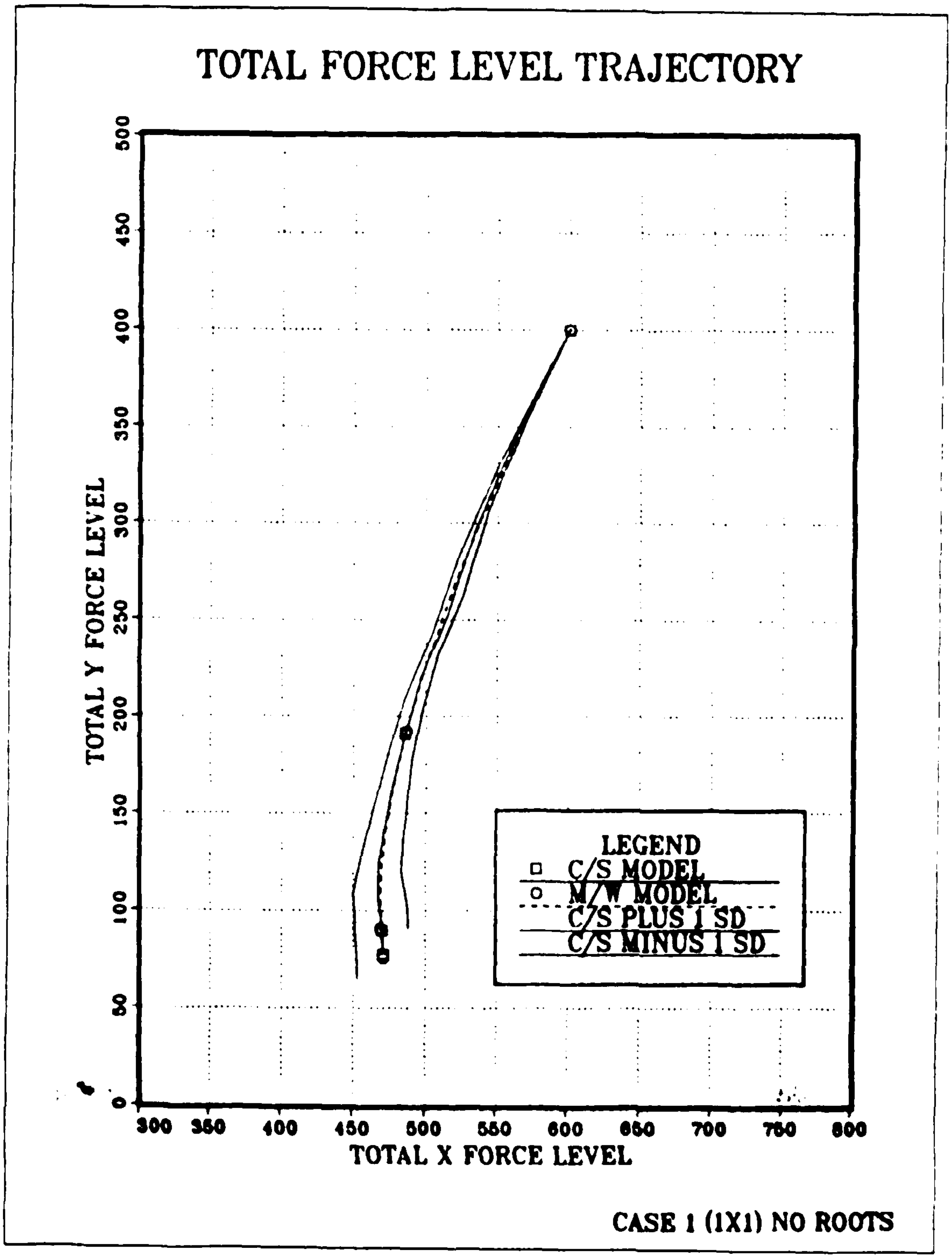

Figure E.1 Total Force Level Trajectory For Case One.

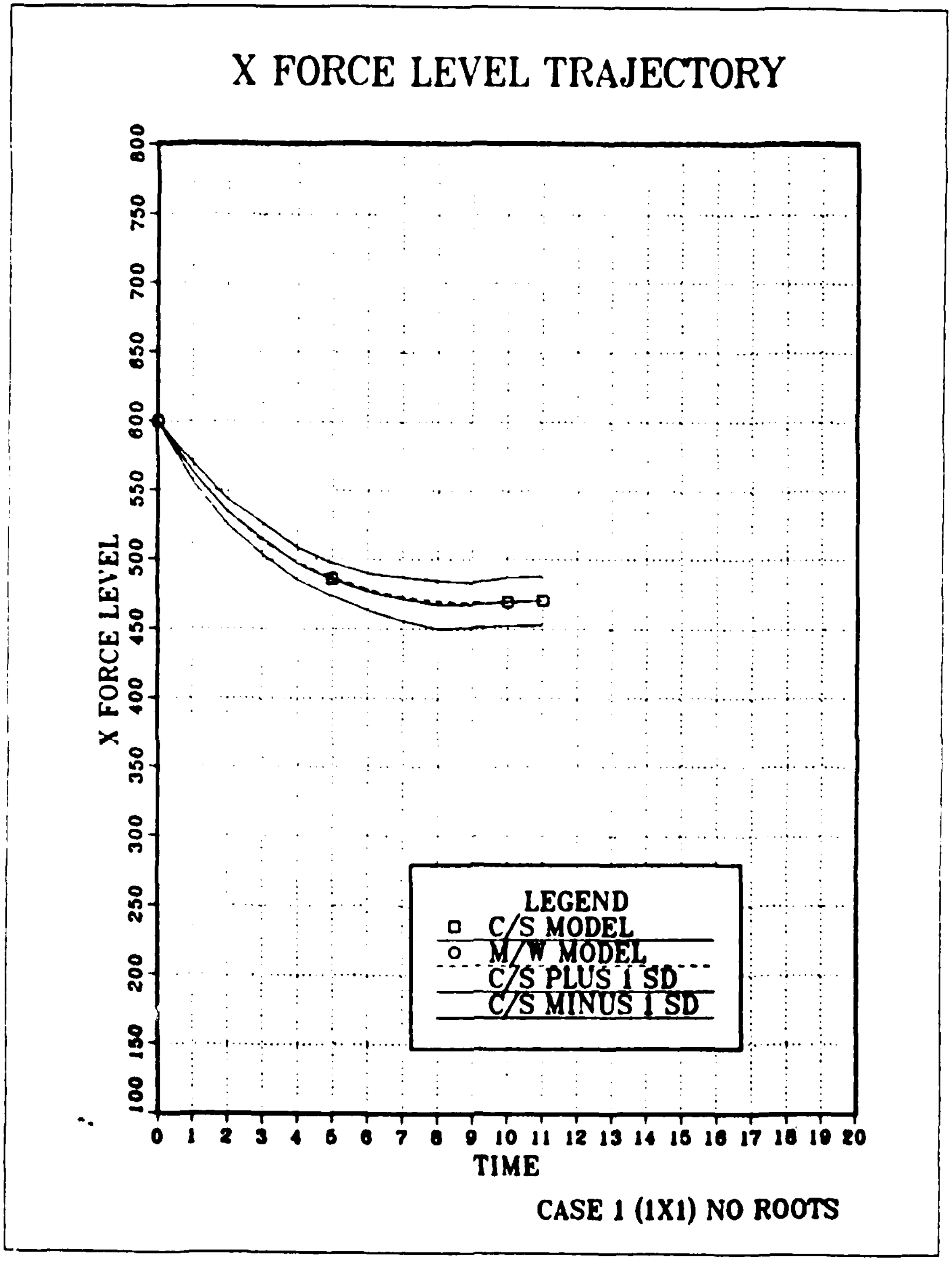

Figure E.2 X Force Level Trajectory Over Time For Case One.

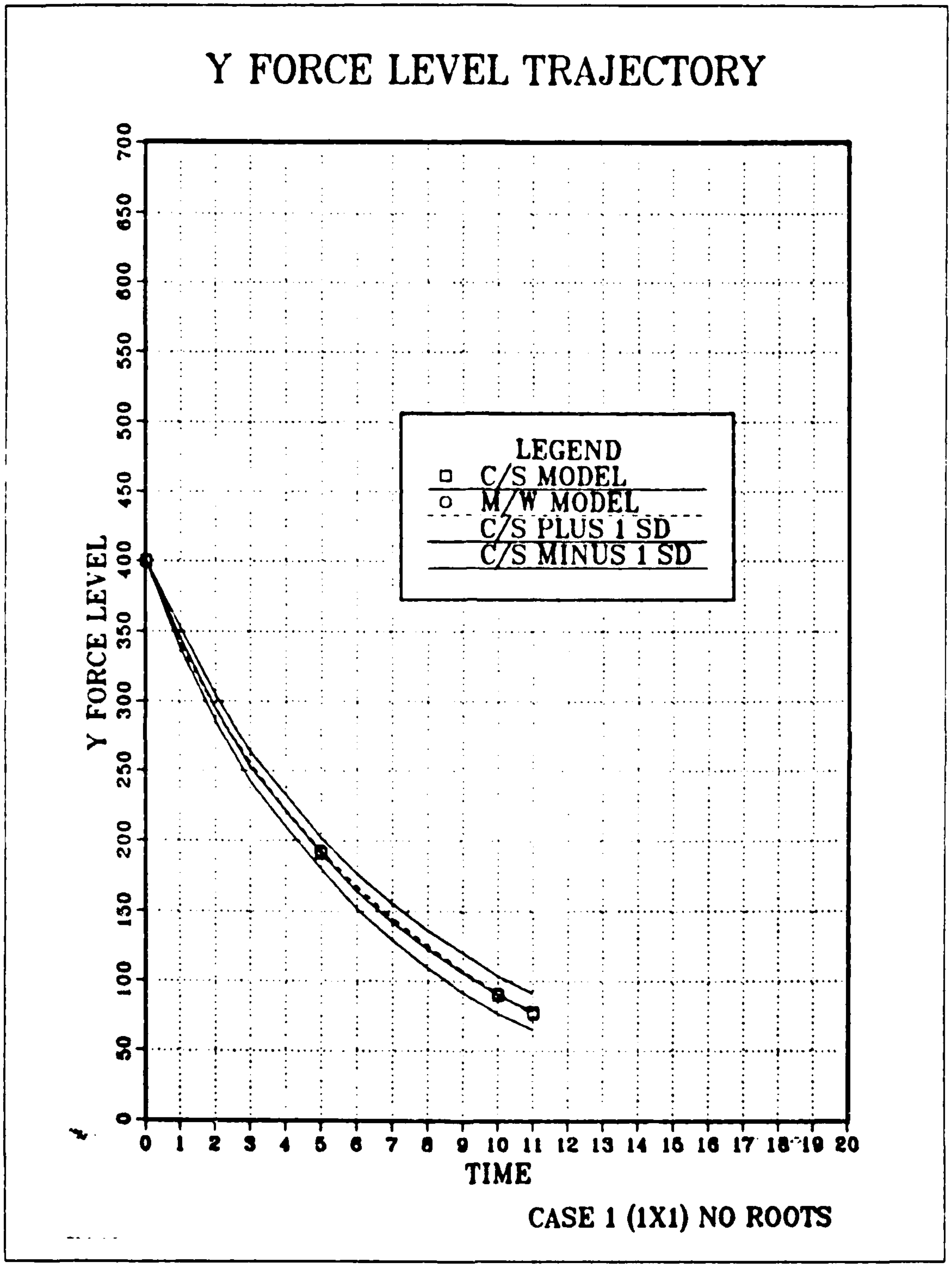

Figure E.3 Y Force Level Trajectory Over Time For Case One.

TABLE 5

INPUT DATA SET FOR C/S MODEL CASE 2

```
01/05/87  TEST OF (1X1), 1 STABLE ROOT AT 200,200S
    99
      1  600    0    0  1.0  1.0  1.0  0.040  0.000  0.000   18    0    0
0.01000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
      1  500    0    0  1.0  1.0  1.0  0.060  0.000  0.000   22    0    0
0.01000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
    345215789.D0    45635761.D0    89342761.D0
```

TABLE 6

INPUT DATA SET FOR M/W MODEL CASE 2

```
01/05/87  TEST OF (1X1), 1 STABLE ROOT AT 200,200S
    99
      1 600.   0.   0.  0.040  0.000  0.000   18    0    0
0.00877 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00018 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
      1 500.   0.   0.  0.060  0.000  0.000   22    0    0
0.00874 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00017 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
```

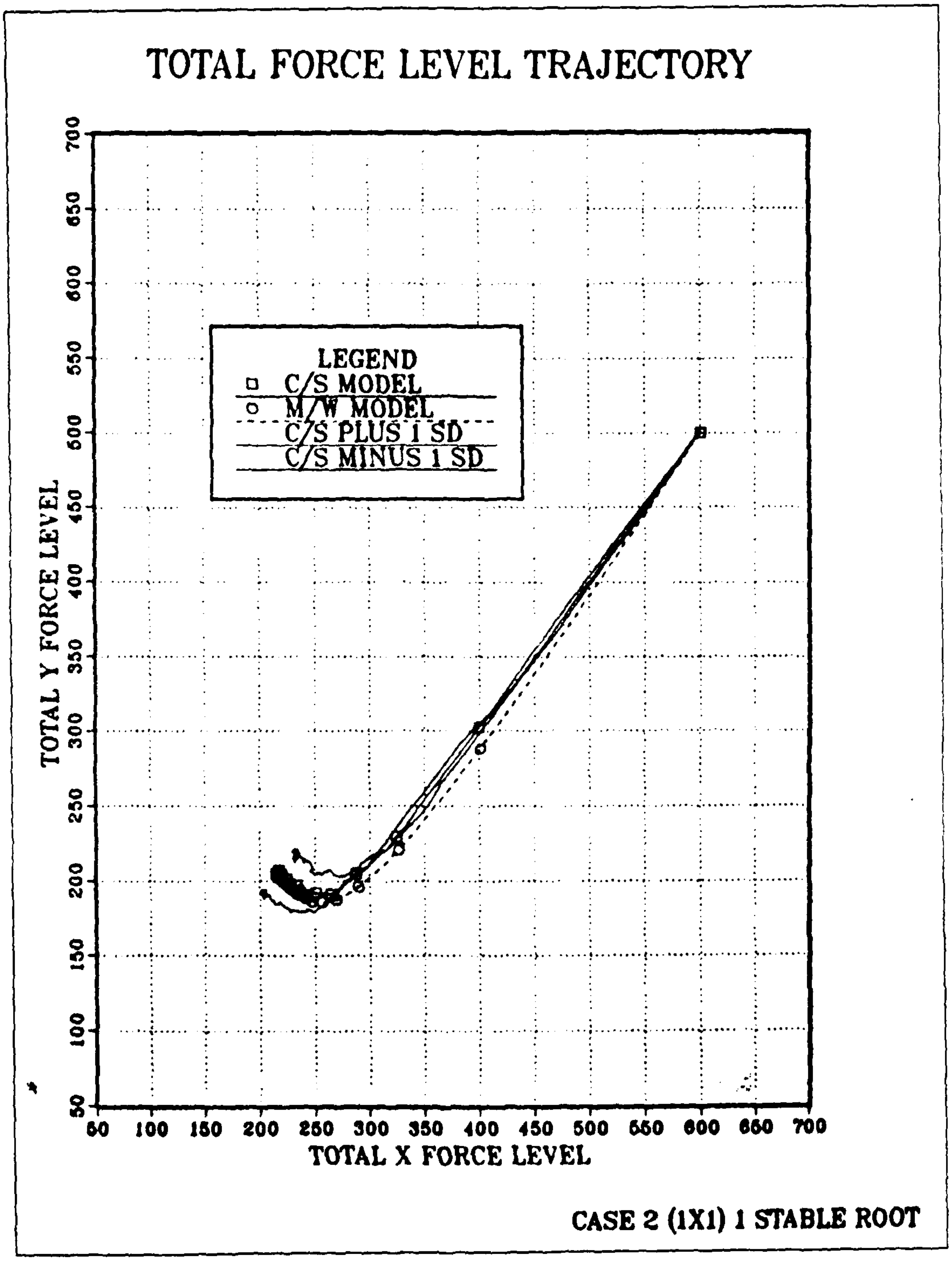

Figure E.4 Total Force Level Trajectory For Case Two.

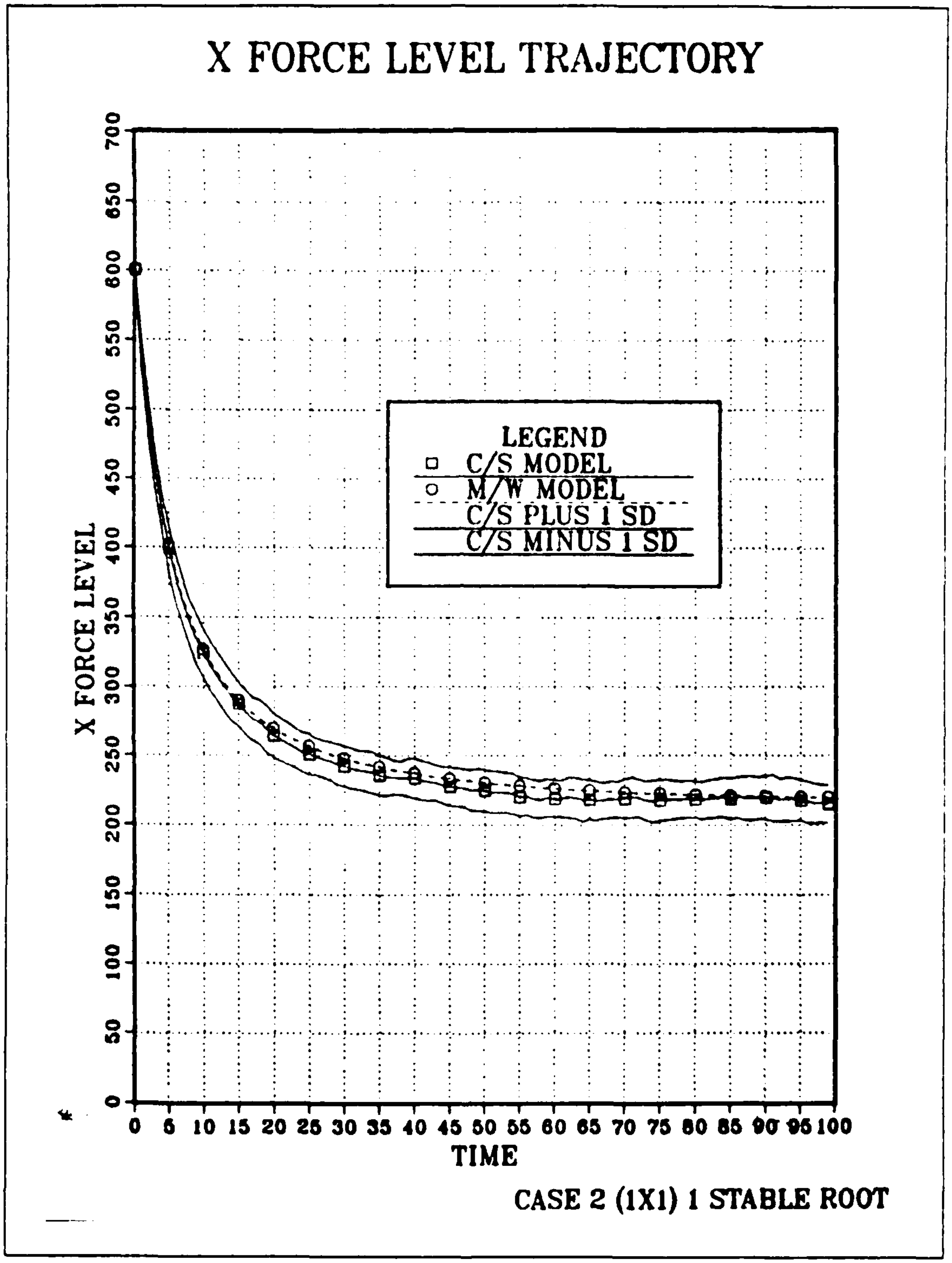

Figure E.5 X Force Level Trajectory Over Time For Case Two.

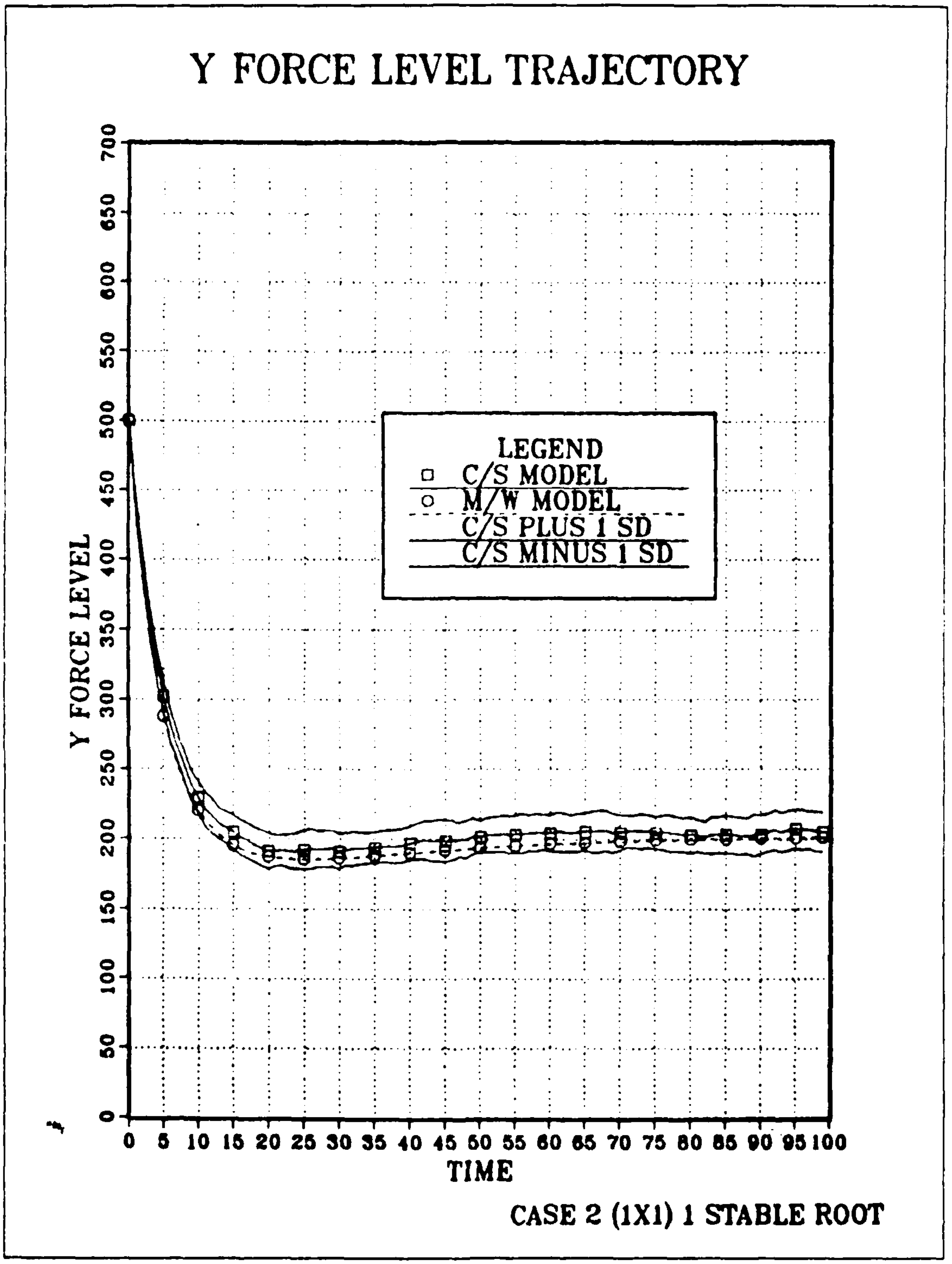

Figure E.6 Y Force Level Trajectory Over Time For Case Two.

TABLE 7

INPUT DATA SET FOR C/S MODEL CASE 3

```
01/06/87  TEST OF (1X1), 1 UNSTABLE ROOT AT 200,200S
 99
   1  500    0    0  1.0  1.0  1.0  0.010  0.000  0.000   22    0    0
0.06000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
   1  600    0    0  1.0  1.0  1.0  0.010  0.000  0.000   18    0    0
0.04000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
   345215789.D0    45635761.D0    89342761.D0
```

TABLE 8

INPUT DATA SET FOR M/W MODEL CASE 3

```
01/06/87  TEST OF (1X1), 1 UNSTABLE ROOT AT 200,200S
 99
   1 500.  0.   0.  0.010  0.000  0.000   22    0    0
0.05129 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00018 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
   1 600.  0.   0.  0.010  0.000  0.000   18    0    0
0.03460 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00017 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
```

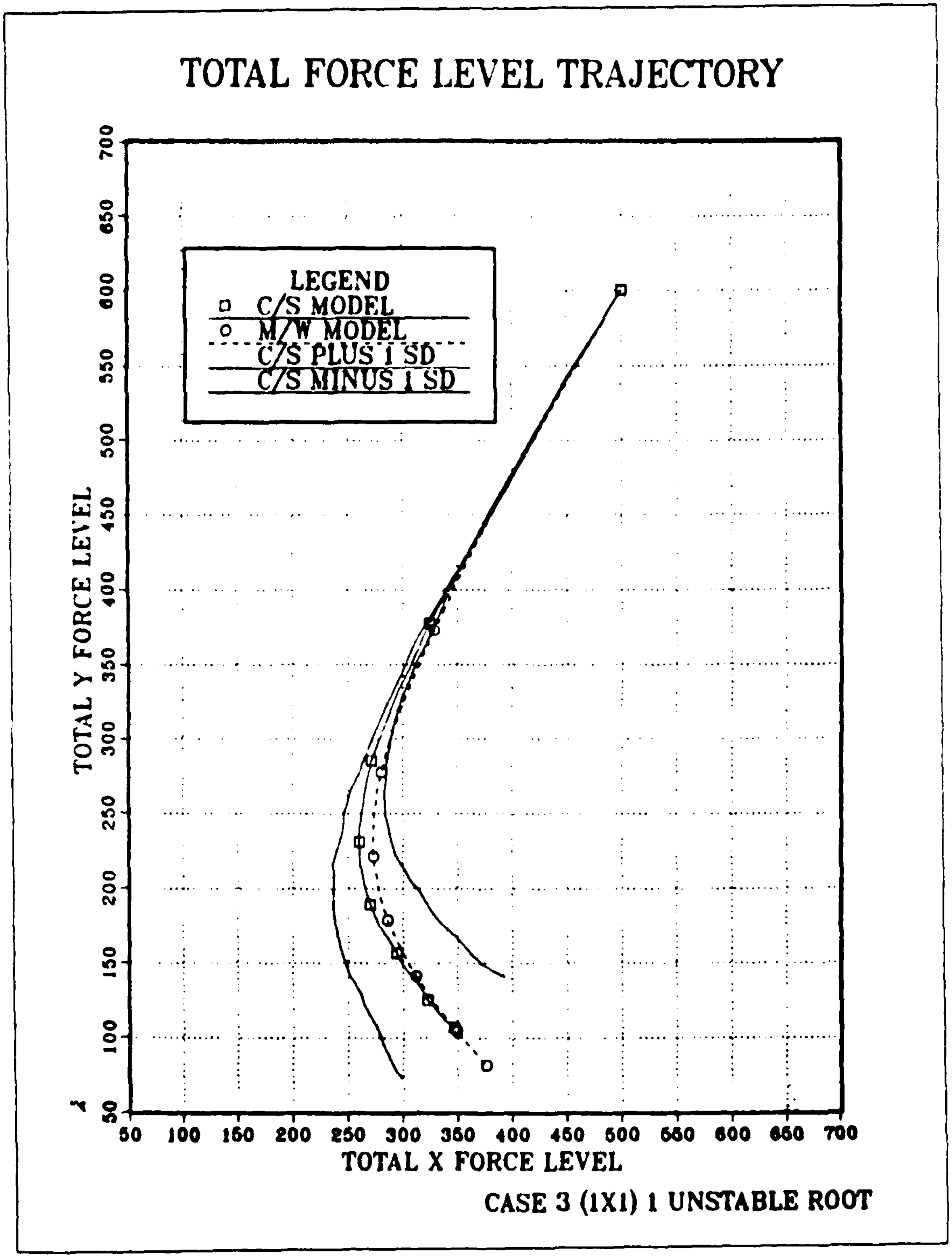

Figure E.7 Total Force Level Trajectory For Case Three.

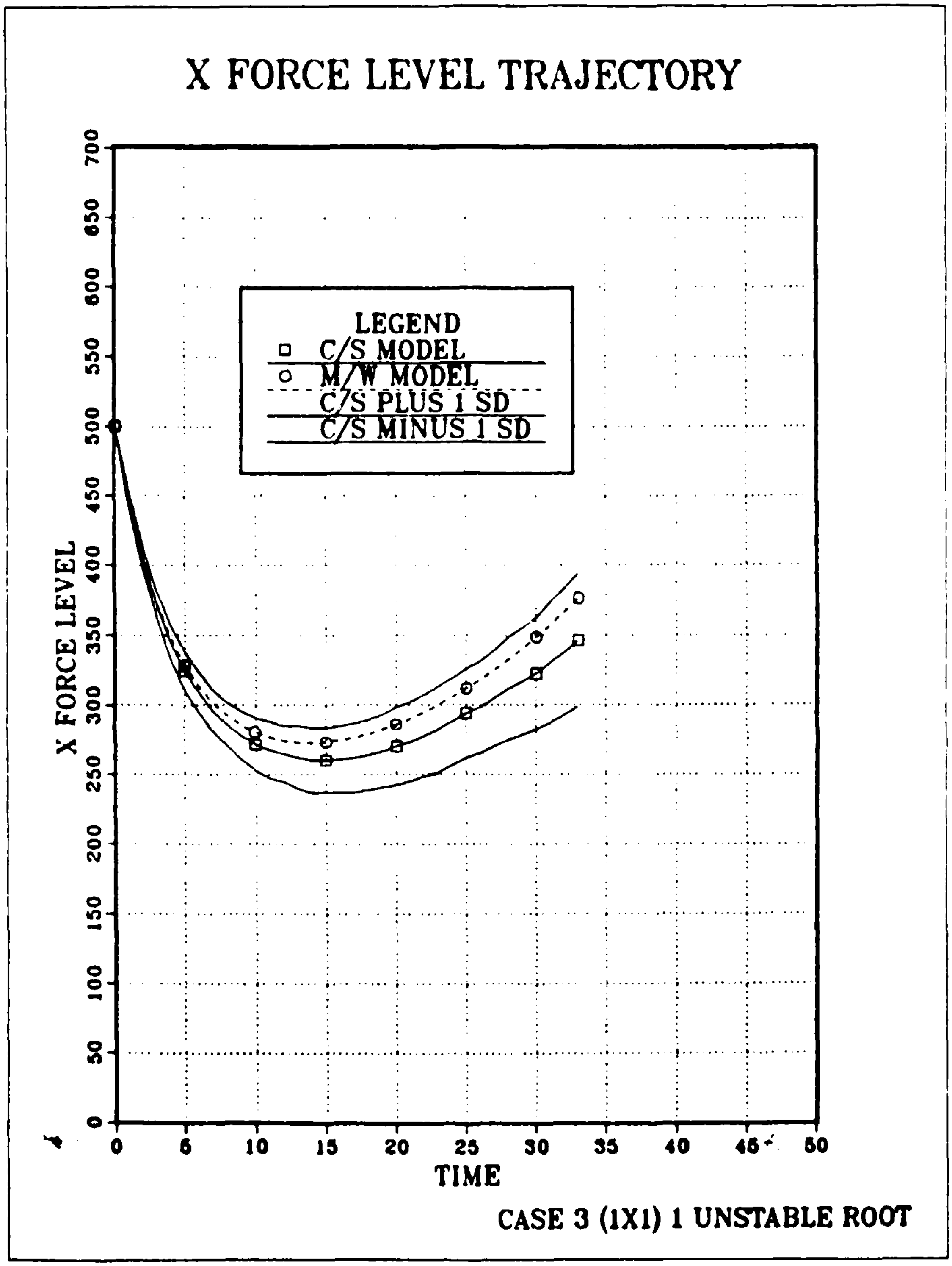

Figure E.8 X Force Level Trajectory Over Time For Case Three.

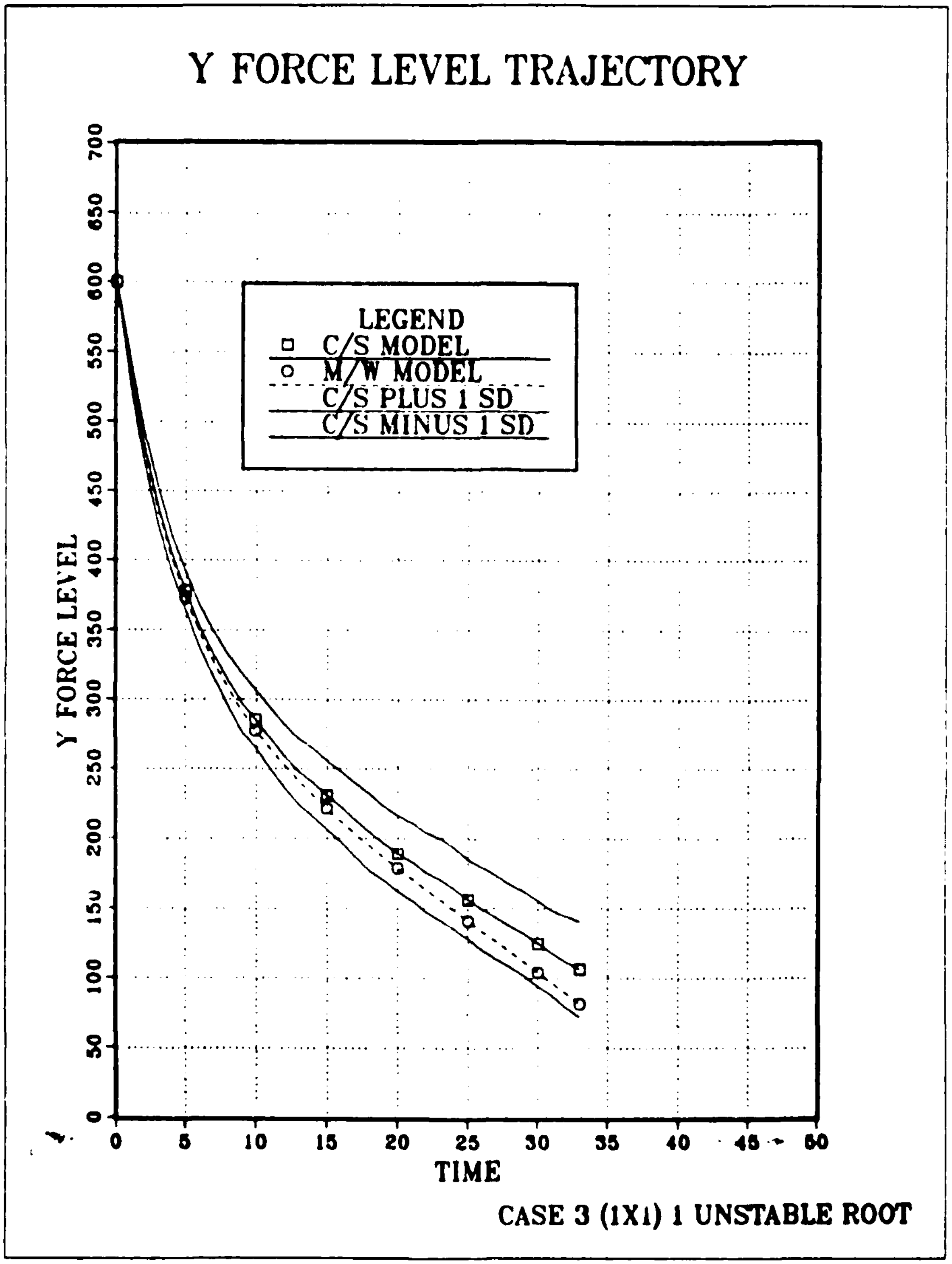

Figure E.9 Y Force Level Trajectory Over Time For Case Three.

TABLE 9

INPUT DATA SET FOR C/S MODEL CASE 4

```
01/05/87  TEST OF (1X1), ROOTS AT 200,200(U) & 366,100(S)S
  99
    1  600    0    0  1.0  1.0  1.0  0.040  0.000  0.000   28    0    0
0.06000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
    1  600    0    0  1.0  1.0  1.0  0.010  0.000  0.000   12    0    0
0.01000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
  345215789.D0    45635761.D0    89342761.D0
```

TABLE 10

INPUT DATA SET FOR M/W MODEL CASE 4

```
01/05/87  TEST OF (1X1), ROOTS AT 200,200(U) & 366,100(S)S
  99
    1 600.   0.   0.  0.040  0.000  0.000   28    0    0
0.04972 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00017 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
    1 600.   0.   0.  0.010  0.000  0.000   12    0    0
0.00796 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00017 0.00000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
```

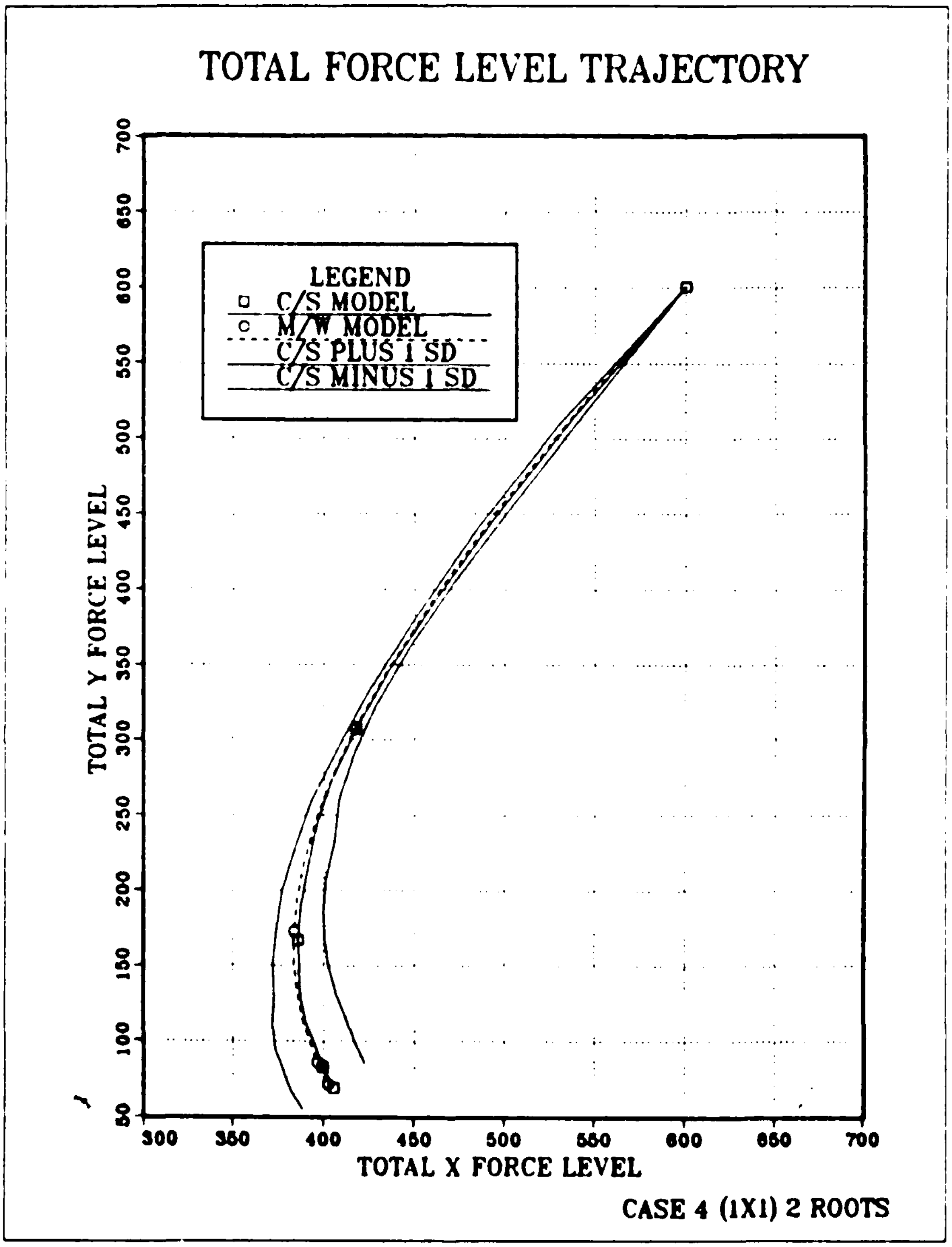

Figure E.10 Total Force Level Trajectory For Case Four.

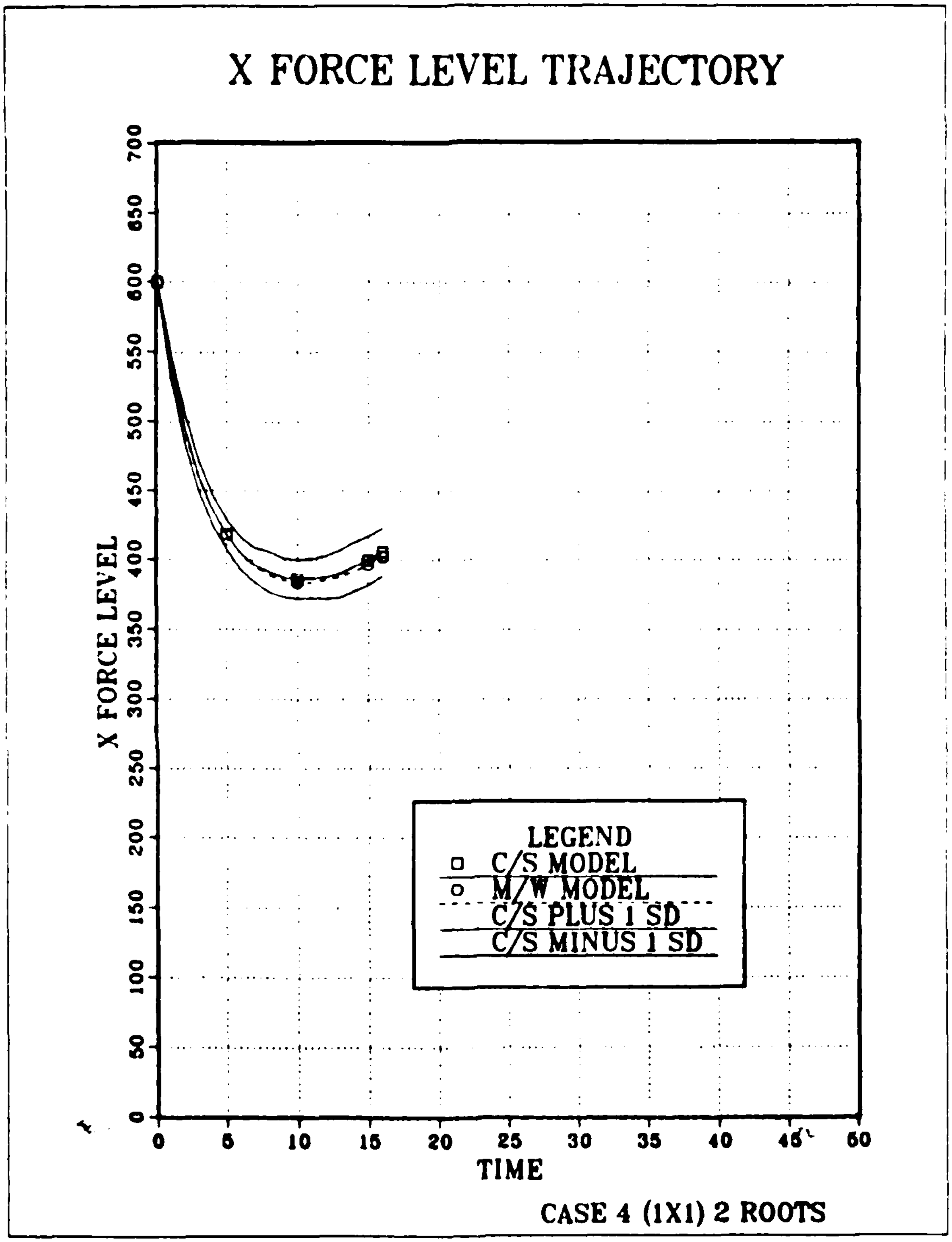

Figure E.11 X Force Level Trajectory Over Time For Case Four.

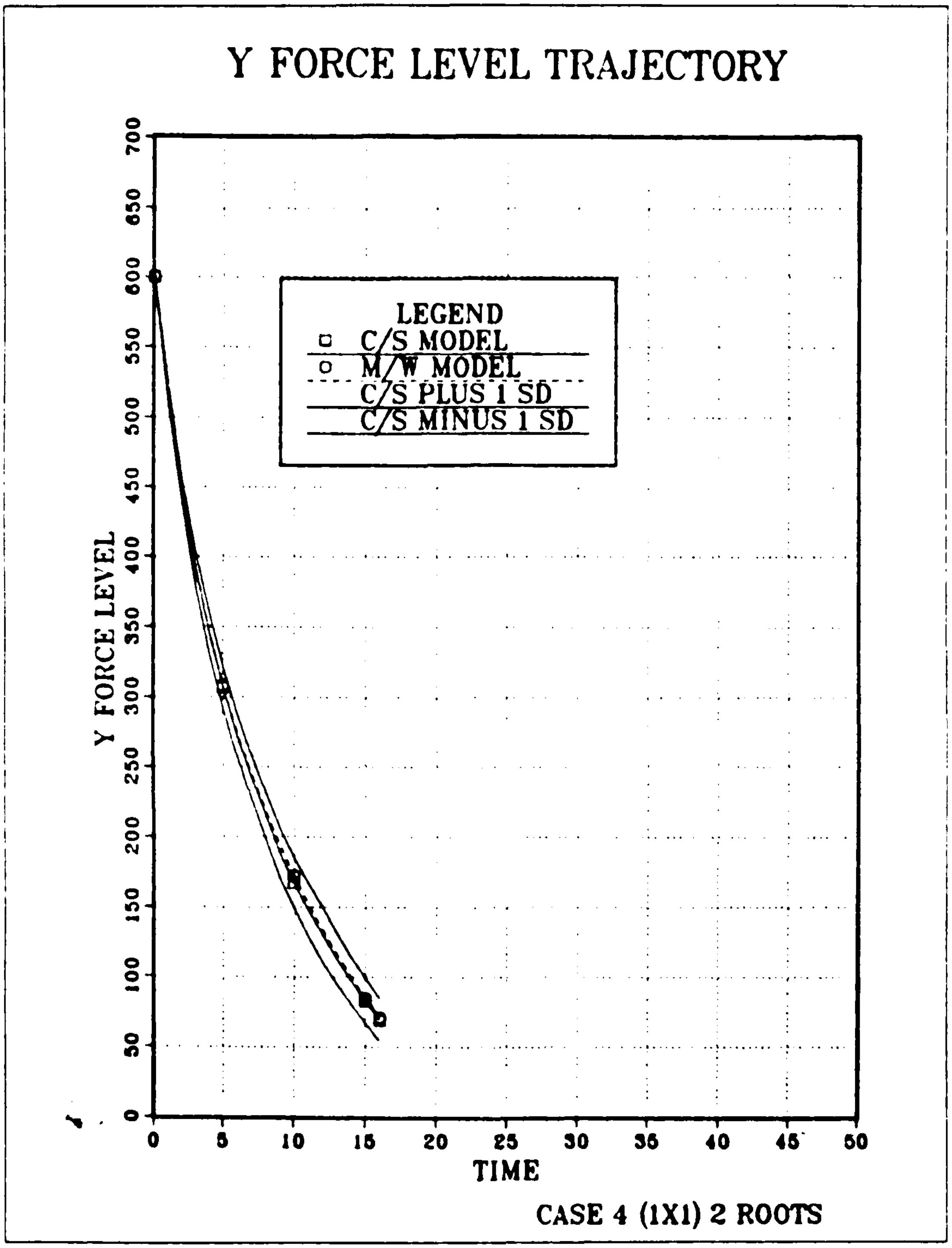

Figure E.12 Y Force Level Trajectory Over Time For Case Four.

TABLE 11

INPUT DATA SET FOR C/S MODEL CASE 5

```
01/31/87  TEST OF (1X2), CASE 5
    99
     1  500  000    0  1.0  1.0  1.0  0.030  0.000  0.000   38    0    0
0.16000 0.06000 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00050 0.00040 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
     2  250  300    0  1.0  1.0  1.0  0.050  0.100  0.000   44   31    0
0.08000 0.00000 0.00000
0.06000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00000 0.00000
0.00030 0.00000 0.00000
0.00000 0.00000 0.00000
   345215789.D0    45635761.D0    89342761.D0
```

TABLE 12

INPUT DATA SET FOR M/W MODEL CASE 5

```
02/02/87  TEST OF (1X2), CASE 5
    99
     1 500. 000.   0.  0.030  0.000  0.000   38    0    0
0.12243 0.04733 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
0.00038 0.00032 0.00000
0.00000 0.00000 0.00000
0.00000 0.00000 0.00000
     2 250. 300.   0.  0.050  0.100  0.000   44   31    0
0.06156 0.00000 0.00000
0.04530 0.00000 0.00000
0.00000 0.00000 0.00000
0.00015 0.00000 0.00000
0.00023 0.00000 0.00000
0.00000 0.00000 0.00000
```

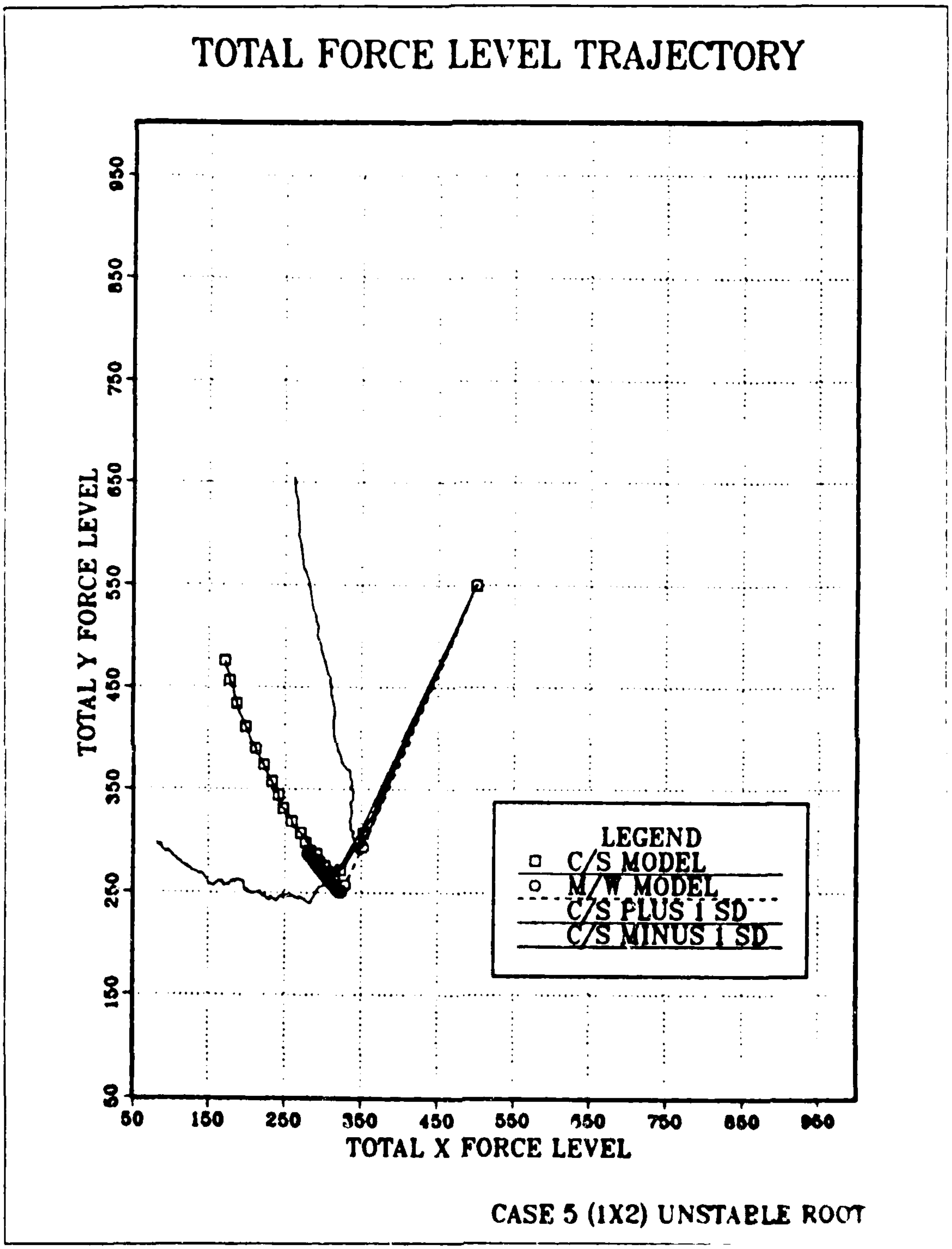

Figure E.13 Total Force Level Trajectory

AD-A183 358 A COMPARATIVE ANALYSIS OF A GENERALIZED LANCHESTER EQUATION MODEL AND A STOCHASTIC COMPUTER SIMULATION MODEL(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA 2/2

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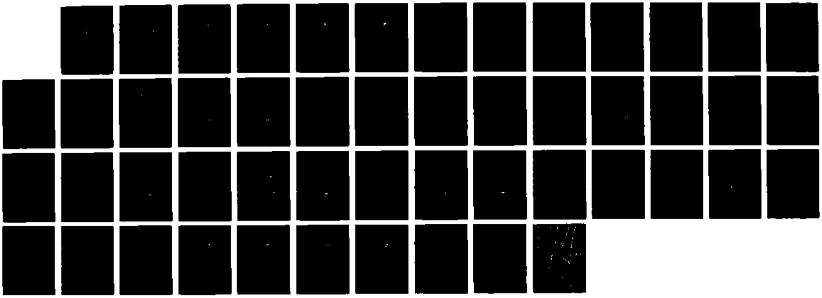

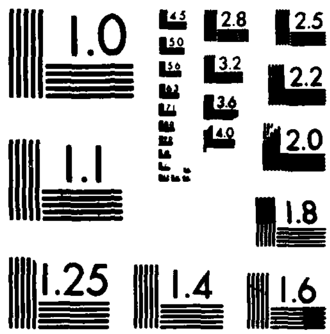

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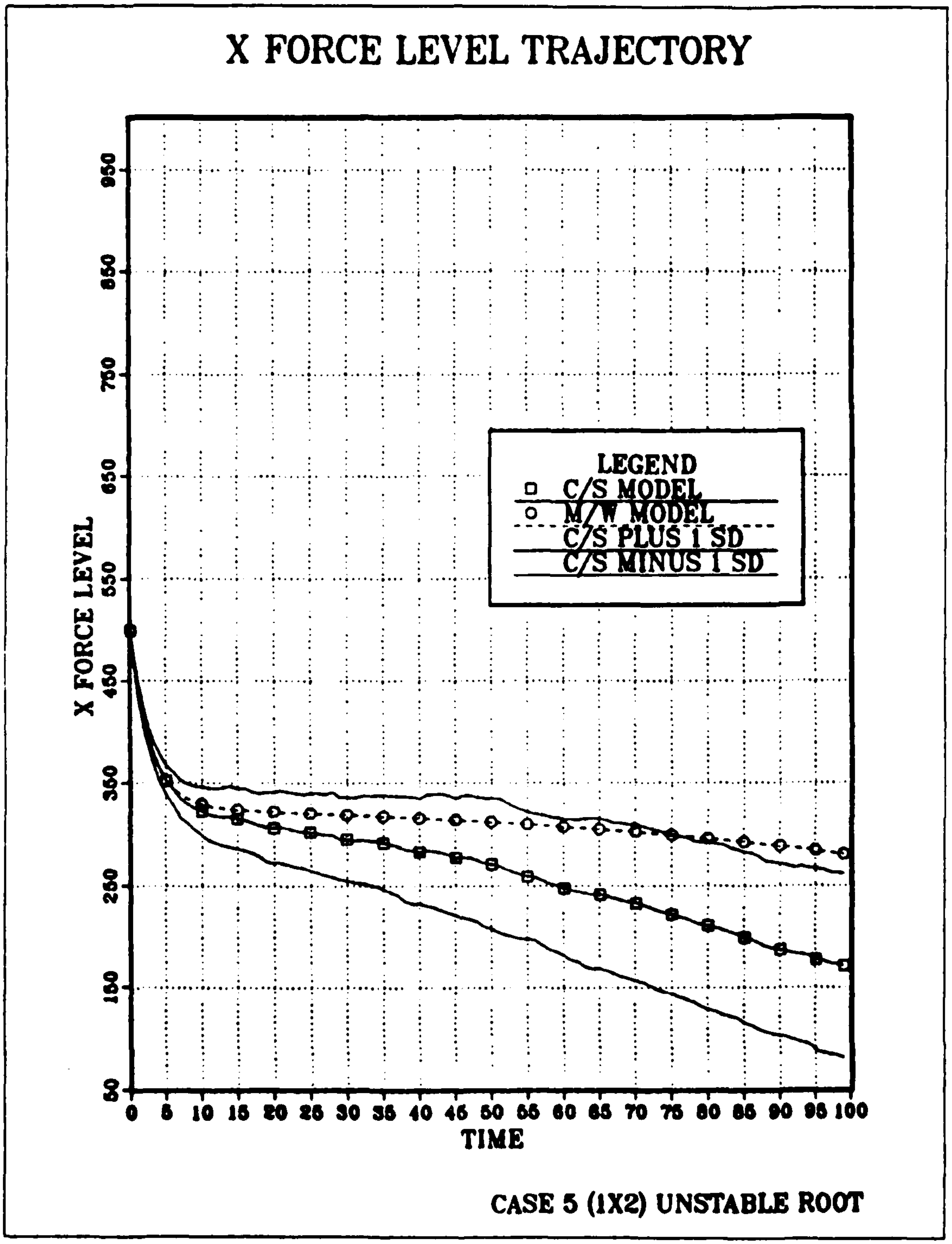

Figure E.14 X Force Level Trajectory Over Time For Case Five.

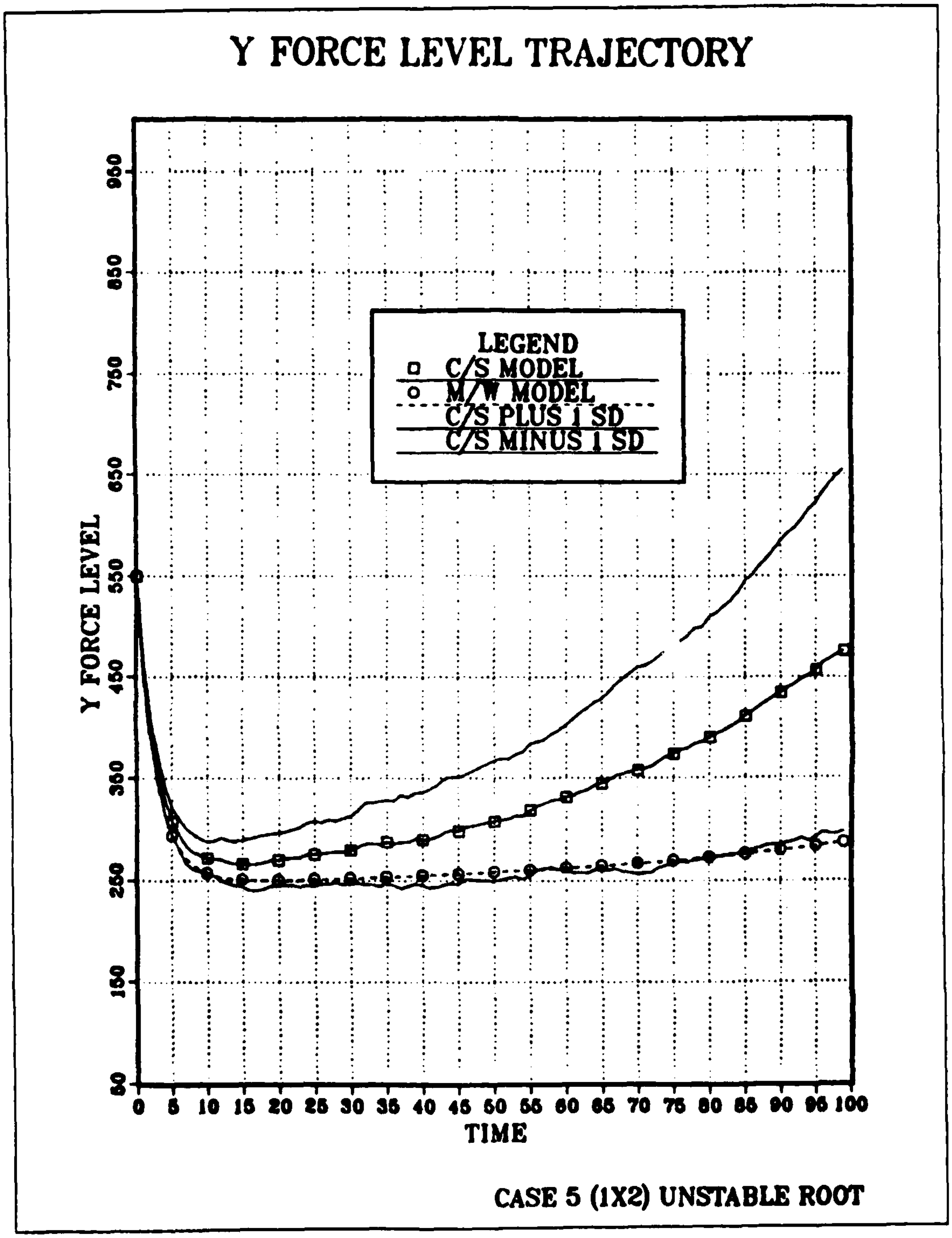

Figure E.15 Y Force Level Trajectory Over Time For Case Five.

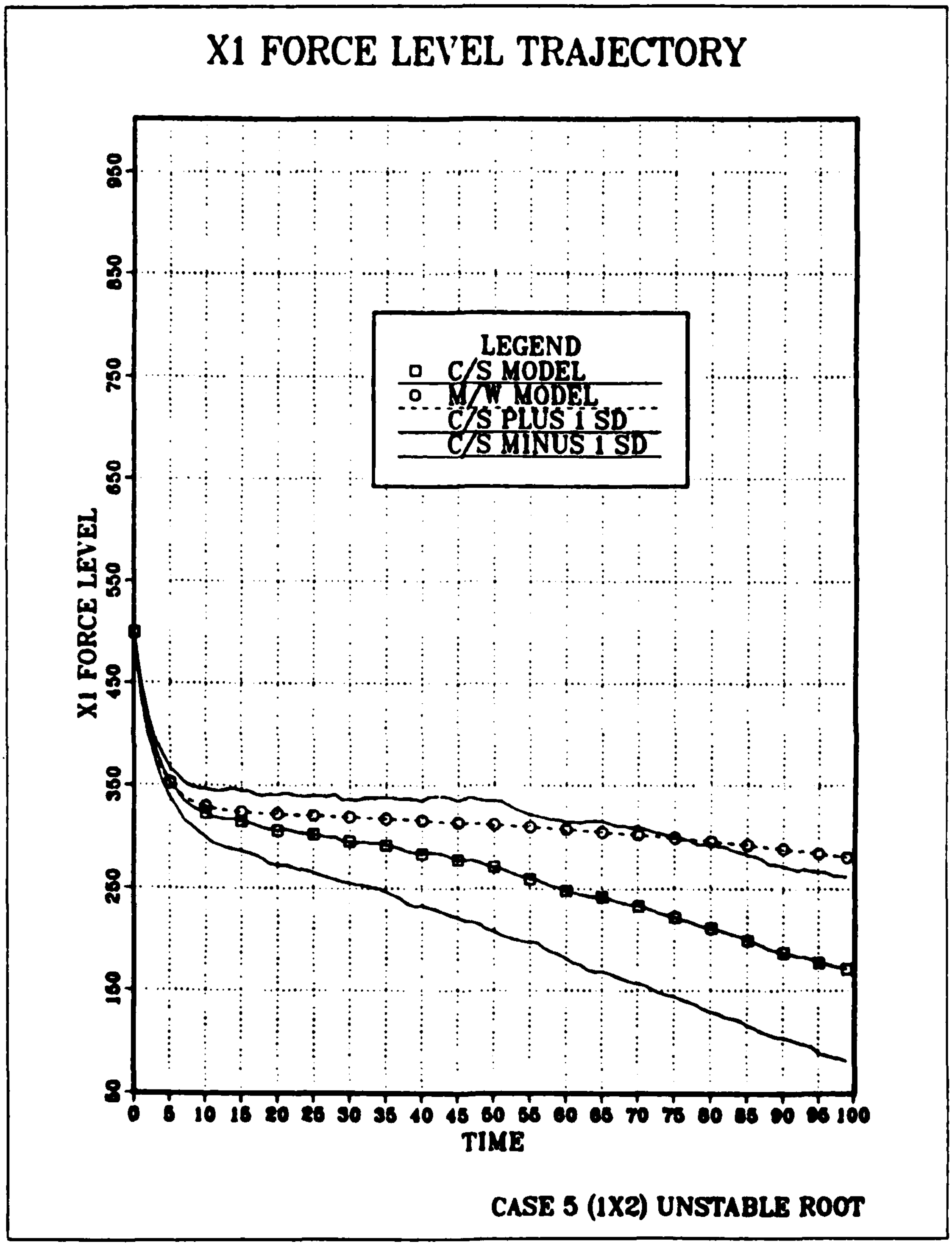

Figure E.16 X1 Force Level Trajectory Over Time For Case Five.

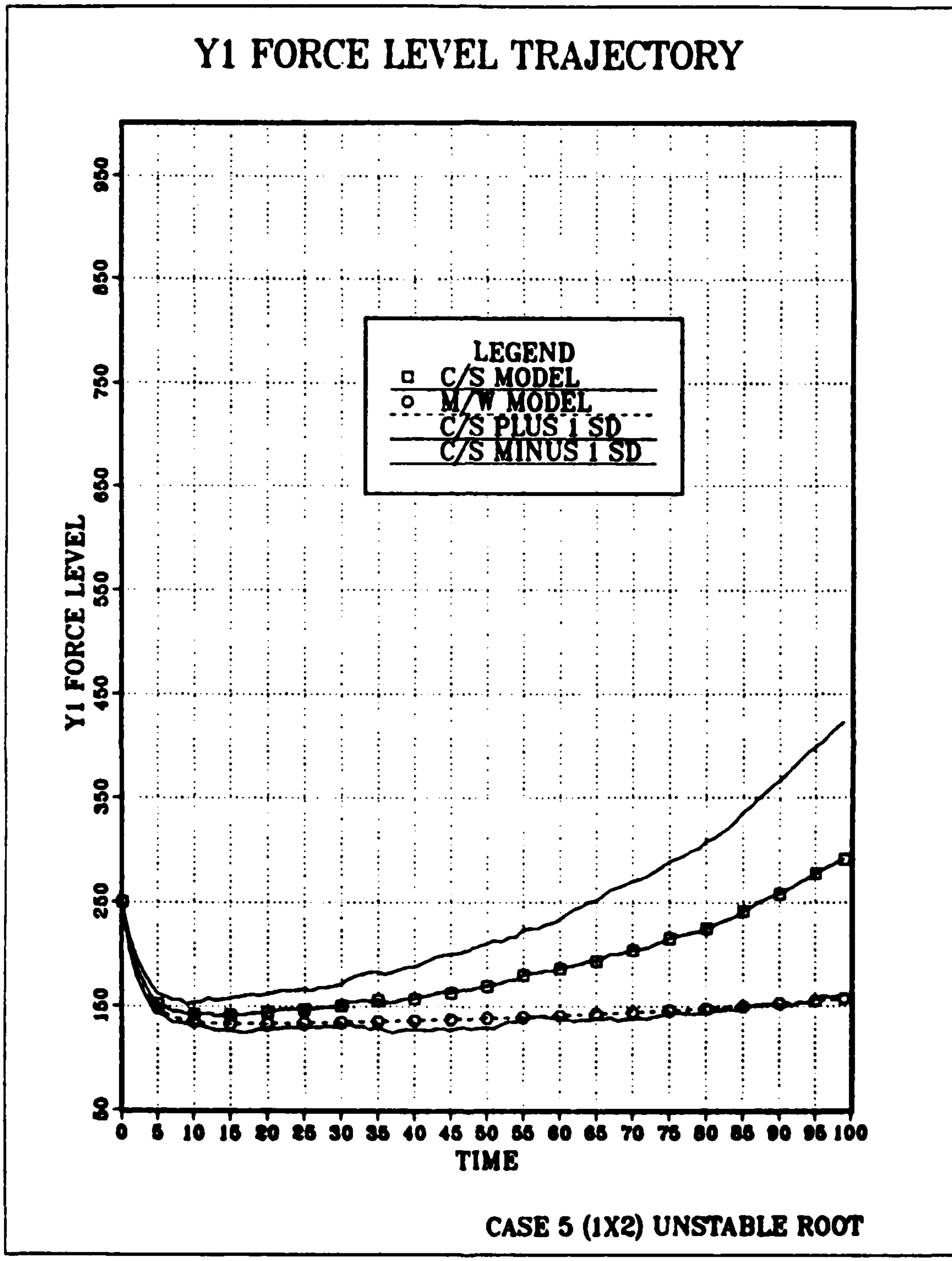

Figure E.17 Y1 Force Level Trajectory Over Time For Case Five.

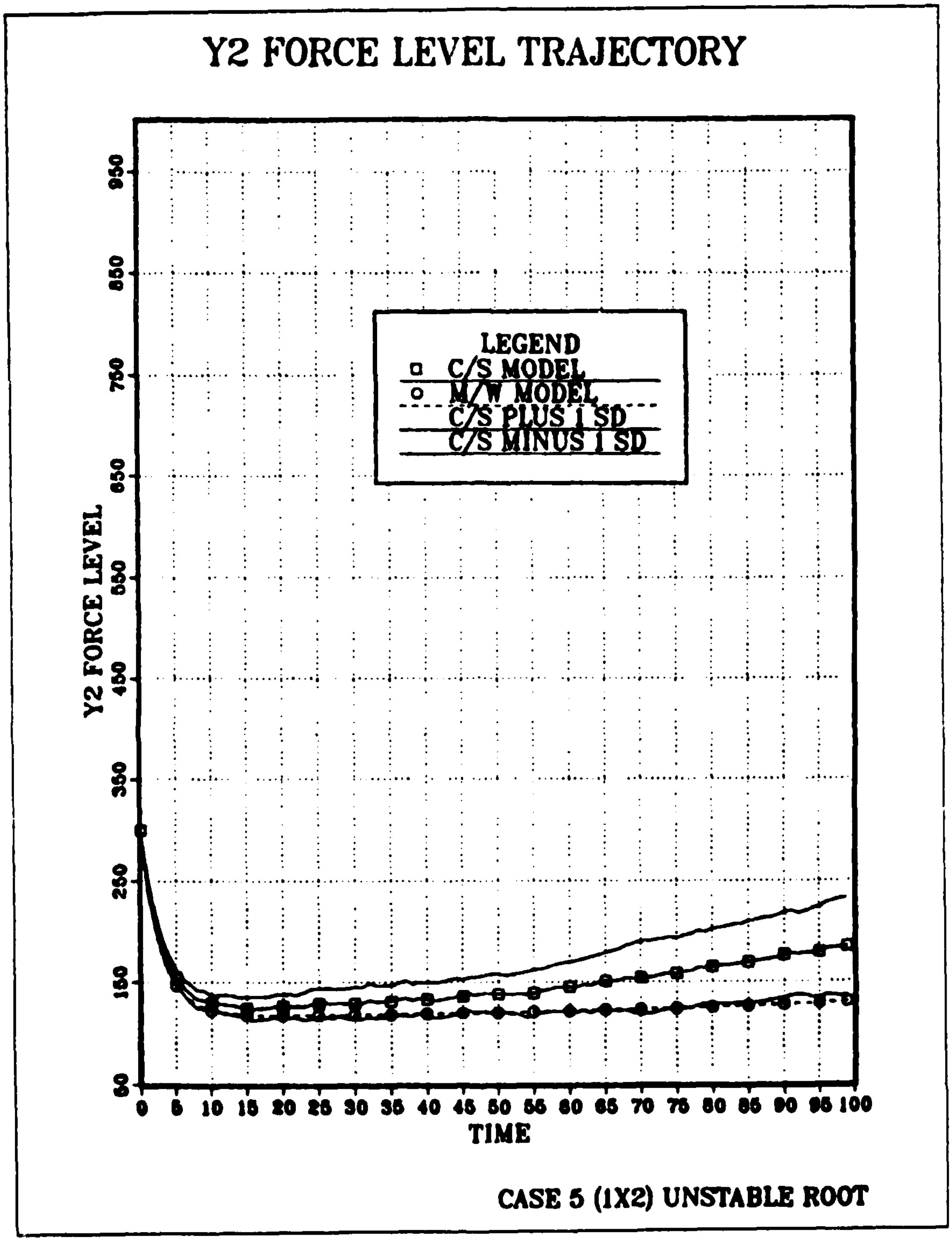

Figure E.18 Y2 Force Level Trajectory Over Time For Case Five.

TABLE 13

INPUT DATA SET FOR C/S MODEL CASE 6

```
01/31/87  TEST OF (2X2), CASE 6, NO ROOTS
   99
     2  200  300    0  1.0  1.0  1.0  0.010  0.020  0.000   11   16    0
0.01000 0.02000 0.00000
0.01000 0.01000 0.00000
0.00000 0.00000 0.00000
0.00008 0.00010 0.00000
0.00010 0.00008 0.00000
0.00000 0.00000 0.00000
     2  300  400    0  1.0  1.0  1.0  0.005  0.010  0.000   15   15    0
0.02000 0.02000 0.00000
0.02000 0.01000 0.00000
0.00000 0.00000 0.00000
0.00010 0.00005 0.00000
0.00005 0.00010 0.00000
0.00000 0.00000 0.00000
   345215789.D0    45635761.D0    89342761.D0
```

TABLE 14

INPUT DATA SET FOR M/W MODEL CASE 6

```
01/31/87  TEST OF (2X2), CASE 6, NO ROOTS
   99
     2 200. 300.   0.  0.010  0.020  0.000   11   16    0
0.00935 0.01828 0.00000
0.00902 0.00925 0.00000
0.00000 0.00000 0.00000
0.00007 0.00009 0.00000
0.00009 0.00007 0.00000
0.00000 0.00000 0.00000
     2 300. 400.   0.  0.005  0.010  0.000   15   15    0
0.01671 0.01683 0.00000
0.01638 0.00823 0.00000
0.00000 0.00000 0.00000
0.00009 0.00004 0.00000
0.00004 0.00008 0.00000
0.00000 0.00000 0.00000
```

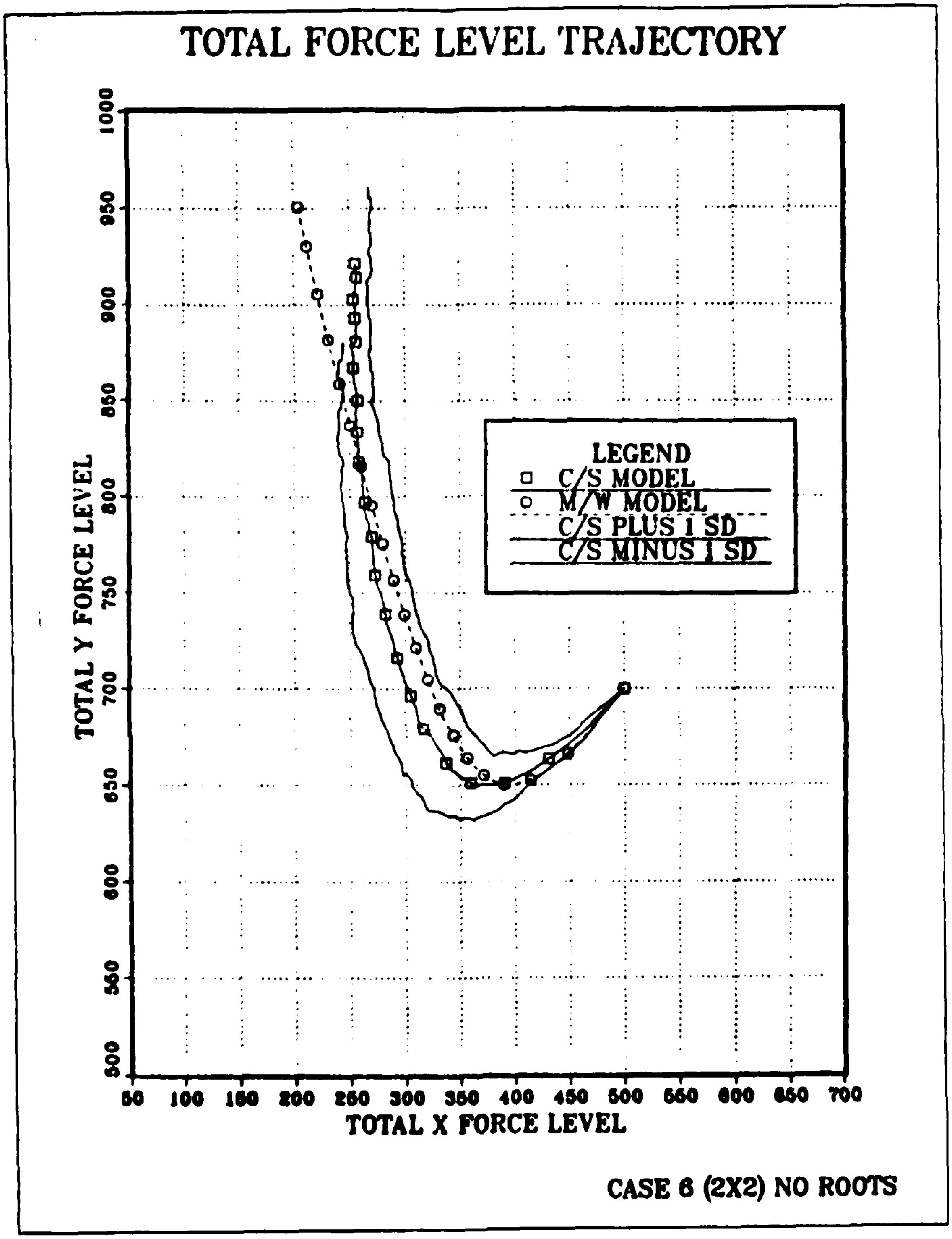

Figure E.19 Total Force Level Trajectory For Case Six.

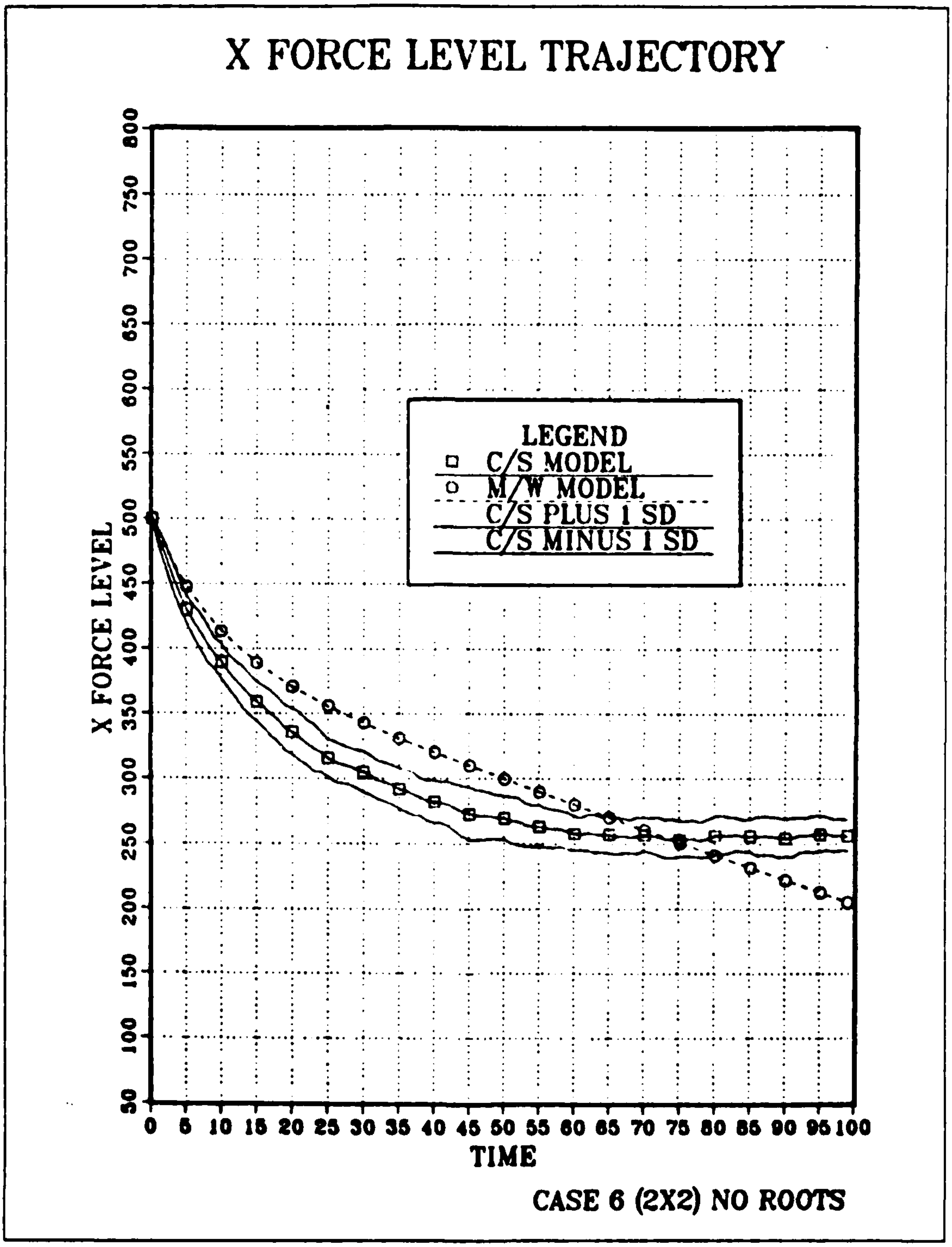

Figure E.20 X Force Level Trajectory Over Time For Case Six.

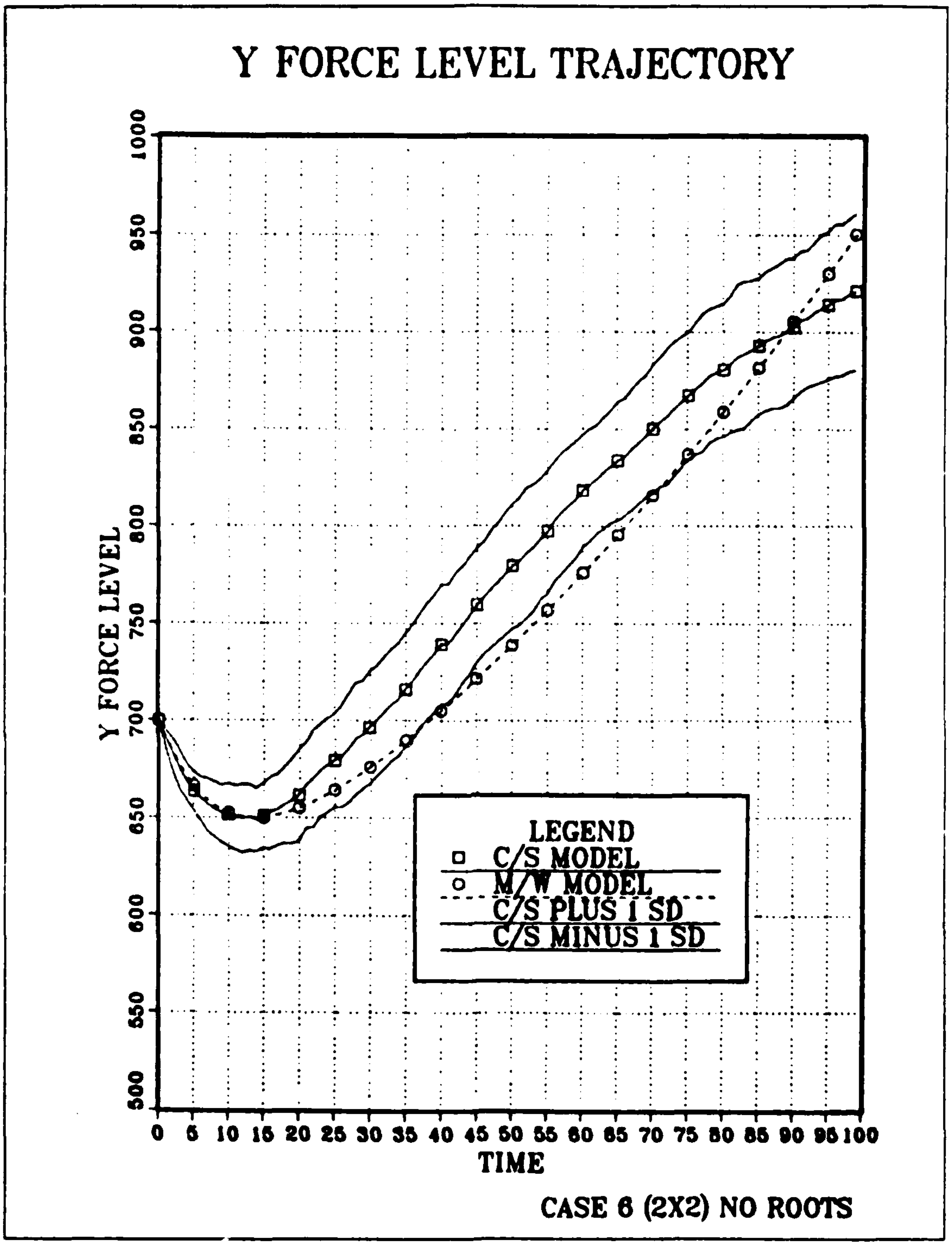

Figure E.21 Y Force Level Trajectory Over Time For Case Six.

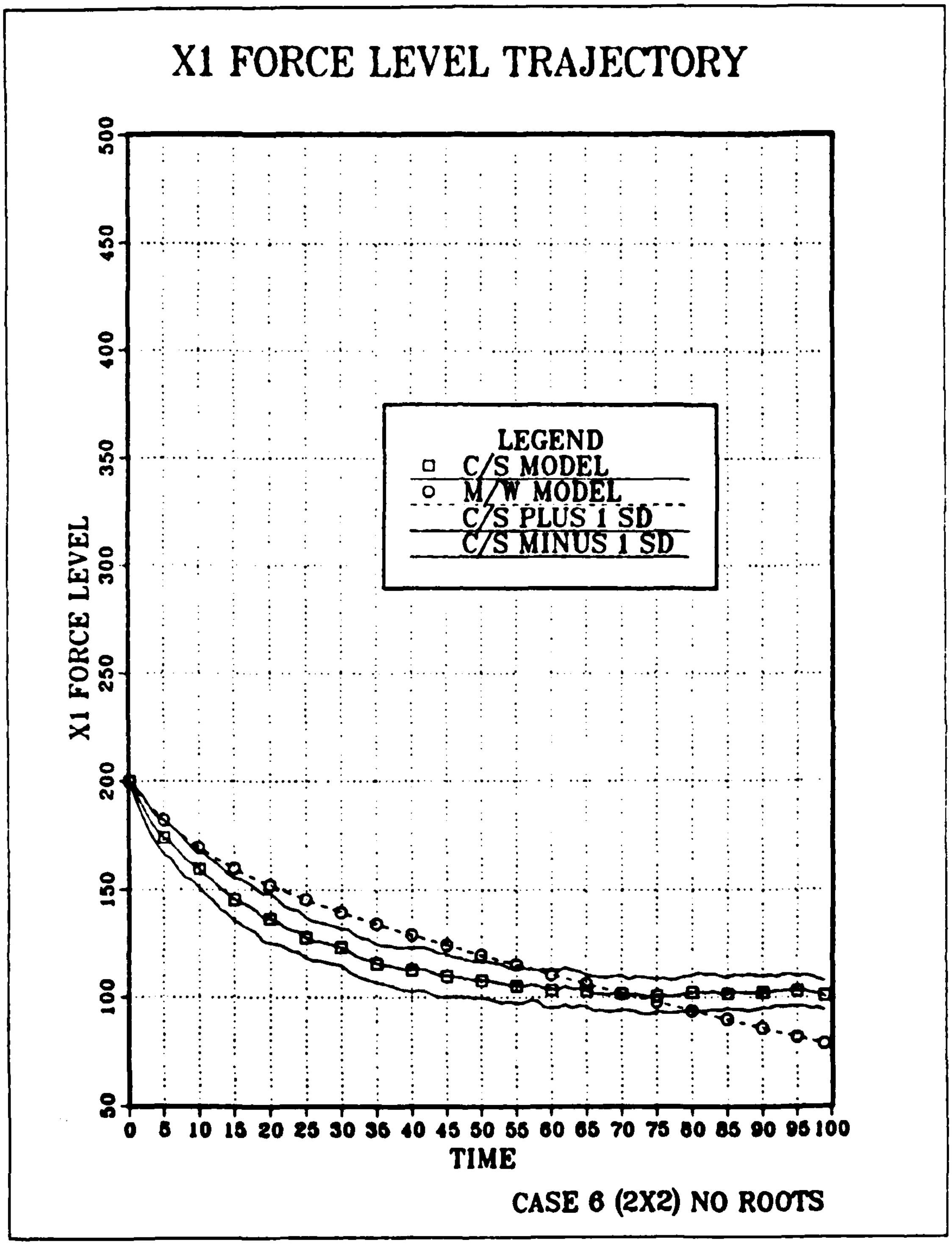

Figure E.22 X1 Force Level Trajectory Over Time For Case Six.

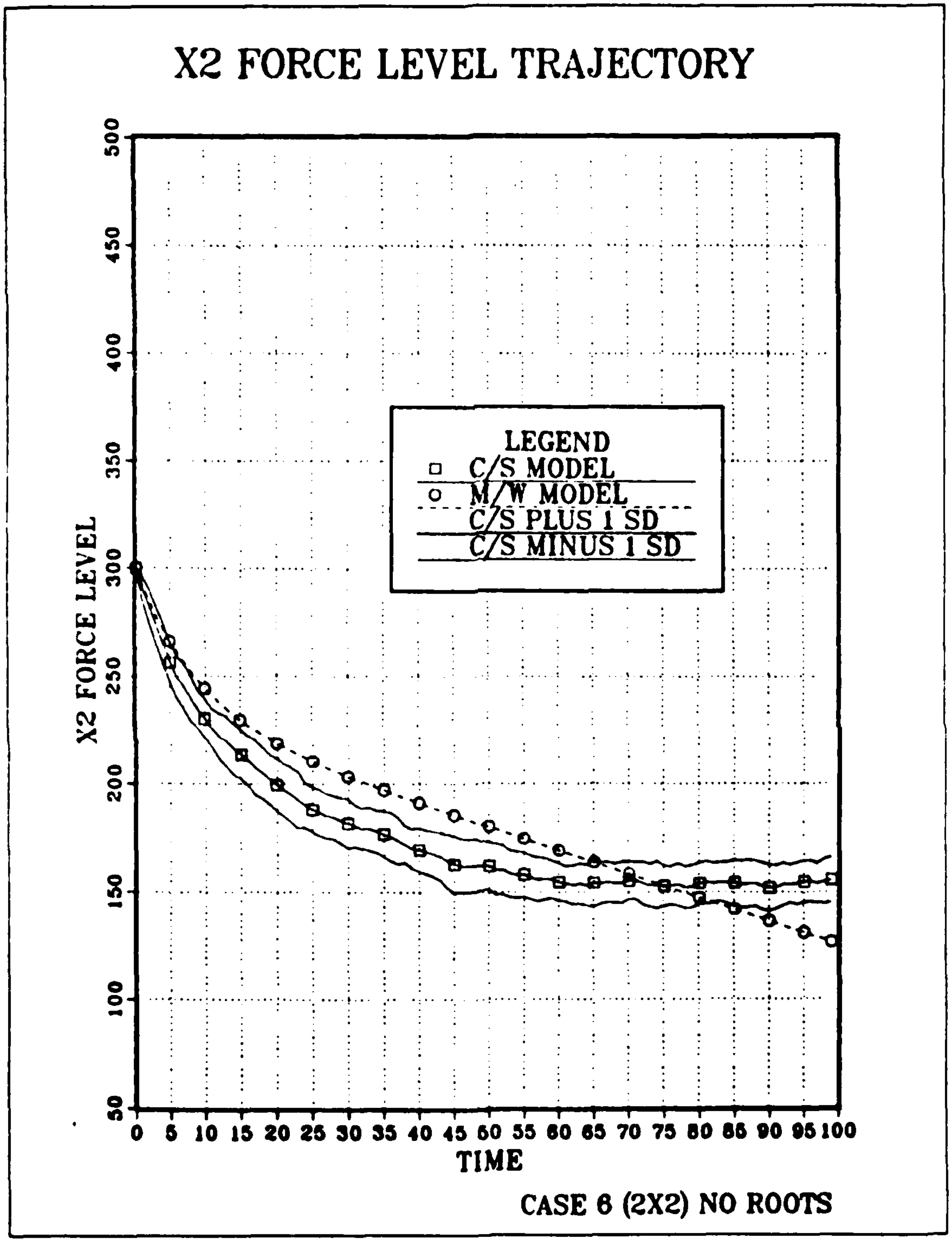

Figure E.23 X2 Force Level Trajectory Over Time For Case Six.

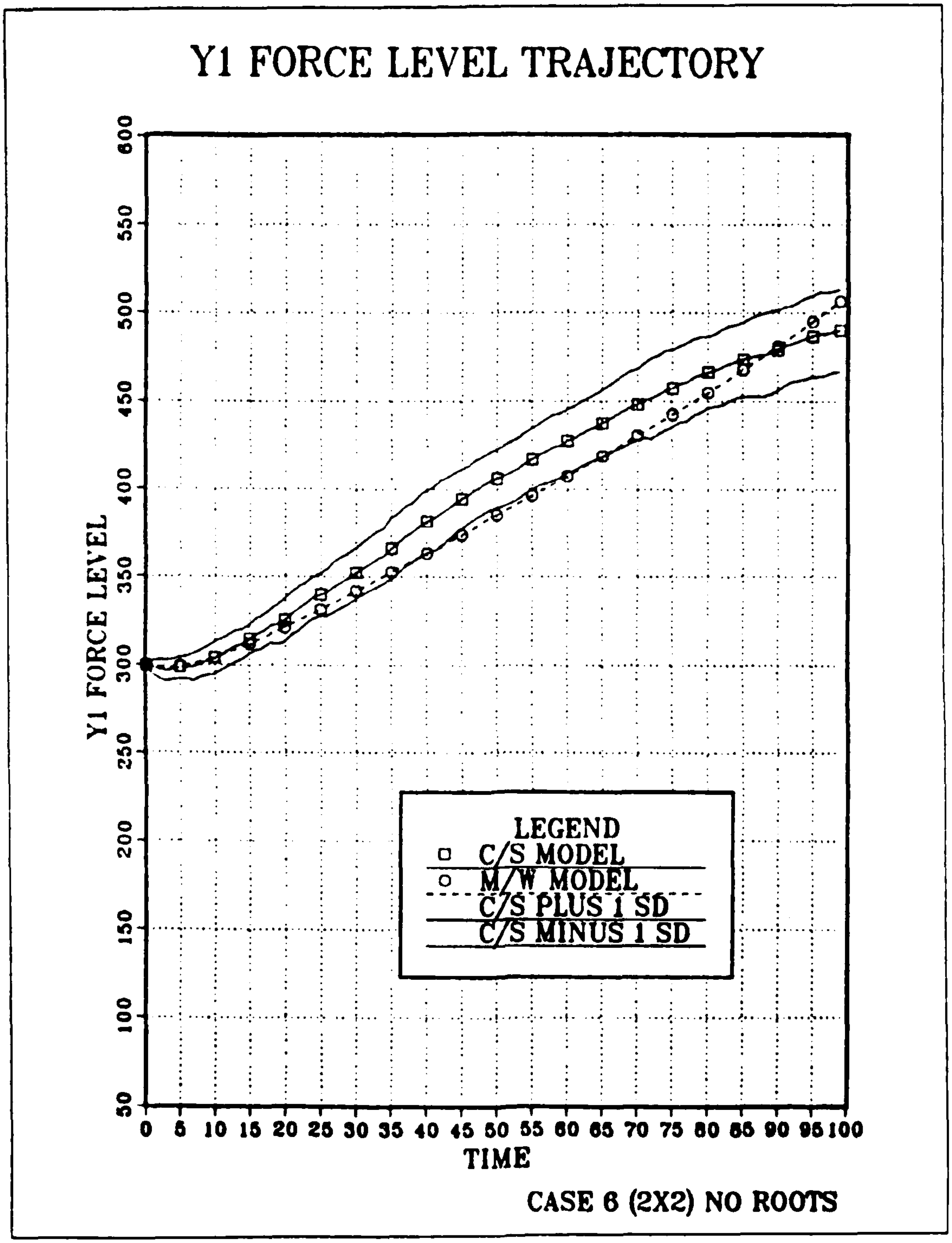

Figure E.24 Y1 Force Level Trajectory Over Time For Case Six.

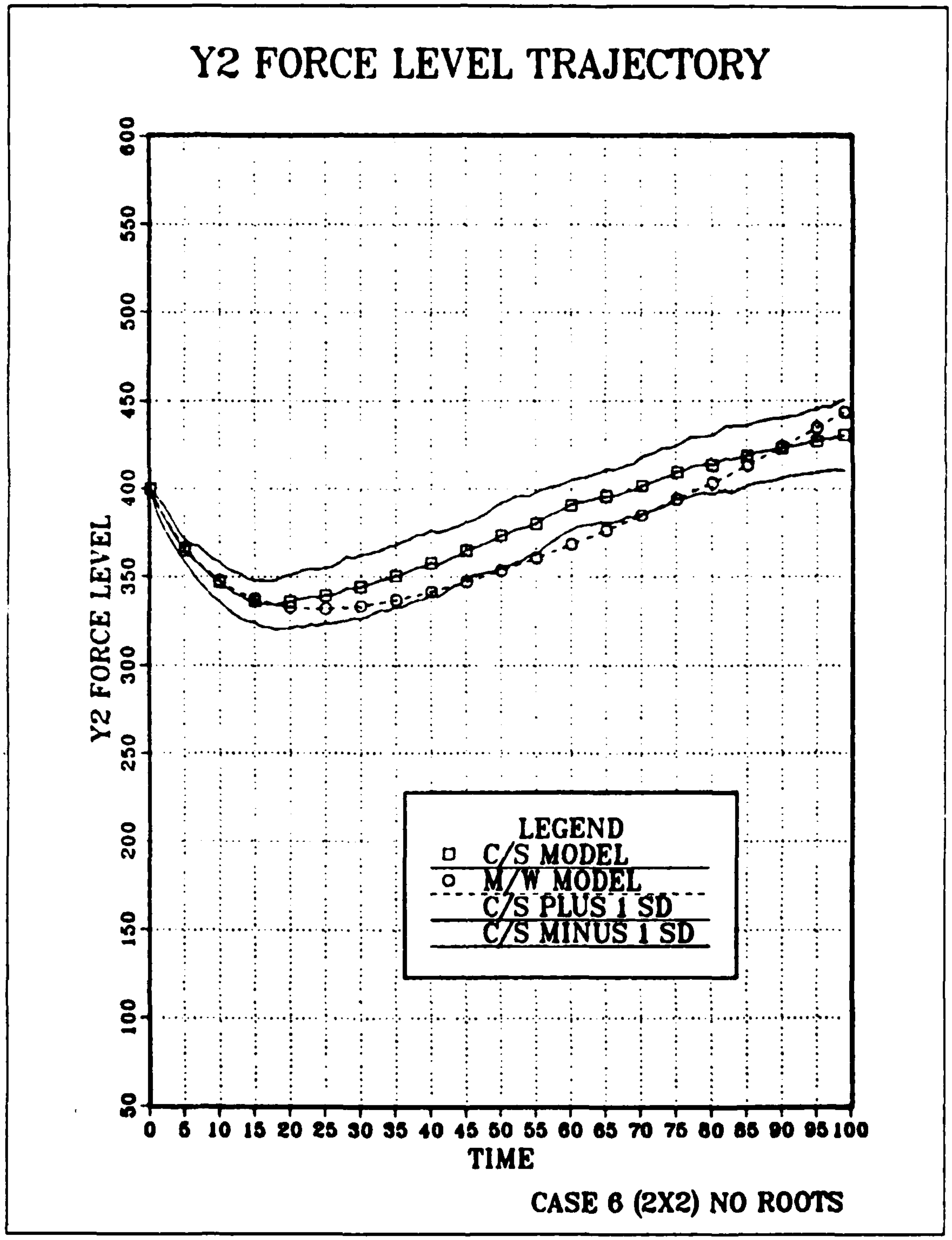

Figure E.25 Y2 Force Level Trajectory Over Time For Case Six.

TABLE 15

INPUT DATA SET FOR C/S MODEL CASE 7

```
01/31/87  TEST OF (2X2), CASE 7, 2 ROOTS
   99
     2  200  300    0  1.0  1.0  1.0  0.010  0.020  0.000   11   16    0
0.01000 0.02000 0.00000
0.01000 0.01000 0.00000
0.00000 0.00000 0.00000
0.00008 0.00010 0.00000
0.00010 0.00008 0.00000
0.00000 0.00000 0.00000
     2  300  400    0  1.0  1.0  1.0  0.005  0.010  0.000    9   12    0
0.02000 0.02000 0.00000
0.02000 0.01000 0.00000
0.00000 0.00000 0.00000
0.00010 0.00005 0.00000
0.00005 0.00010 0.00000
0.00000 0.00000 0.00000
   345215789.D0    45635761.D0    89342761.D0
```

TABLE 16

INPUT DATA SET FOR M/W MODEL CASE 7

```
01/31/87  TEST OF (2X2), CASE 7, 2 ROOTS
   99
     2 200. 300.    0.  0.010  0.020  0.000   11   16    0
0.00944 0.01891 0.00000
0.00942 0.00925 0.00000
0.00000 0.00000 0.00000
0.00007 0.00009 0.00000
0.00009 0.00007 0.00000
0.00000 0.00000 0.00000
     2 300. 400.    0.  0.005  0.010  0.000    9   12    0
0.01900 0.01905 0.00000
0.01821 0.00957 0.00000
0.00000 0.00000 0.00000
0.00009 0.00005 0.00000
0.00005 0.00009 0.00000
0.00000 0.00000 0.00000
```

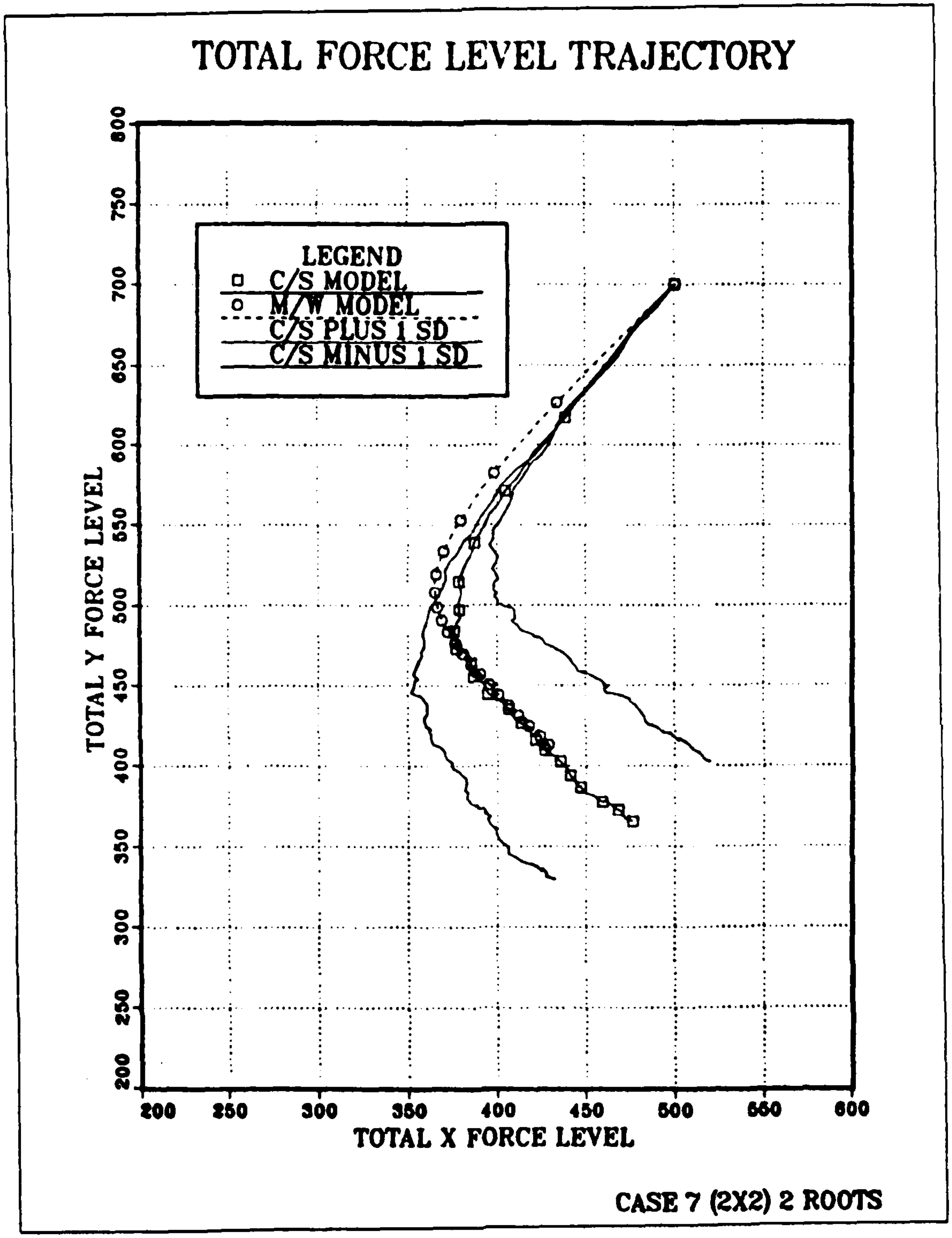

Figure E.26 Total Force Level Trajectory For Case Seven.

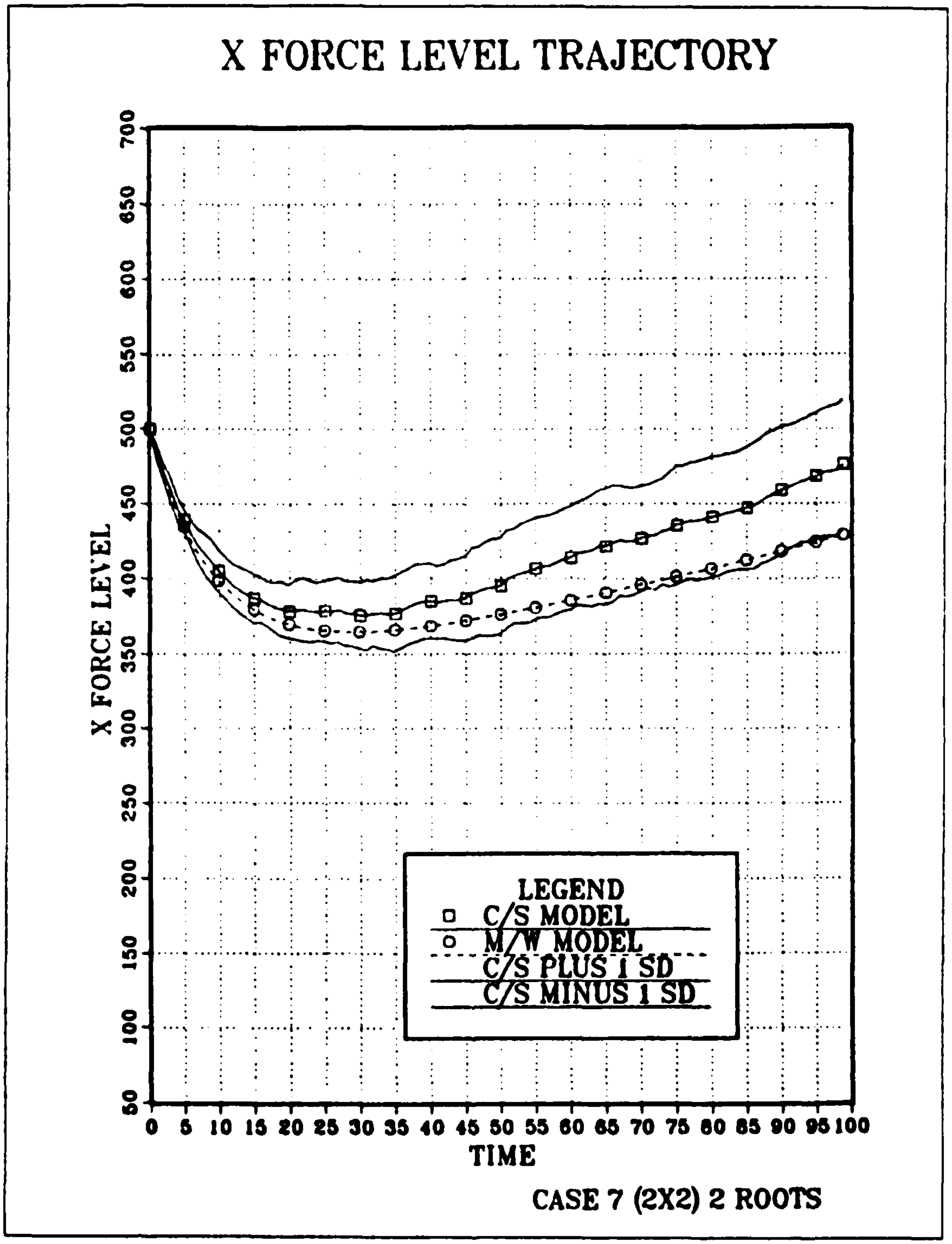

Figure E.27 X Force Level Trajectory Over Time For Case Seven.

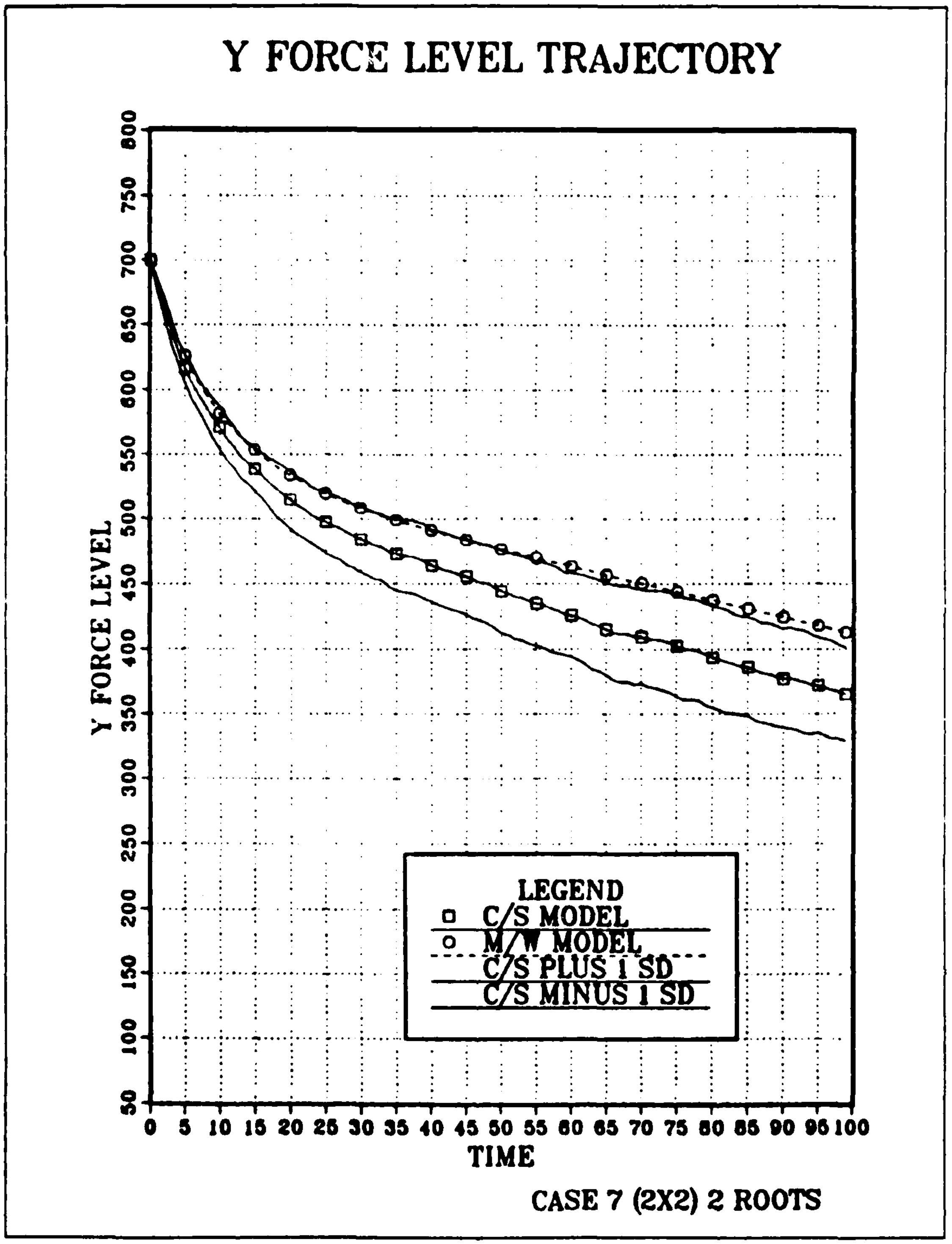

Figure E.28 Y Force Level Trajectory Over Time For Case Seven.

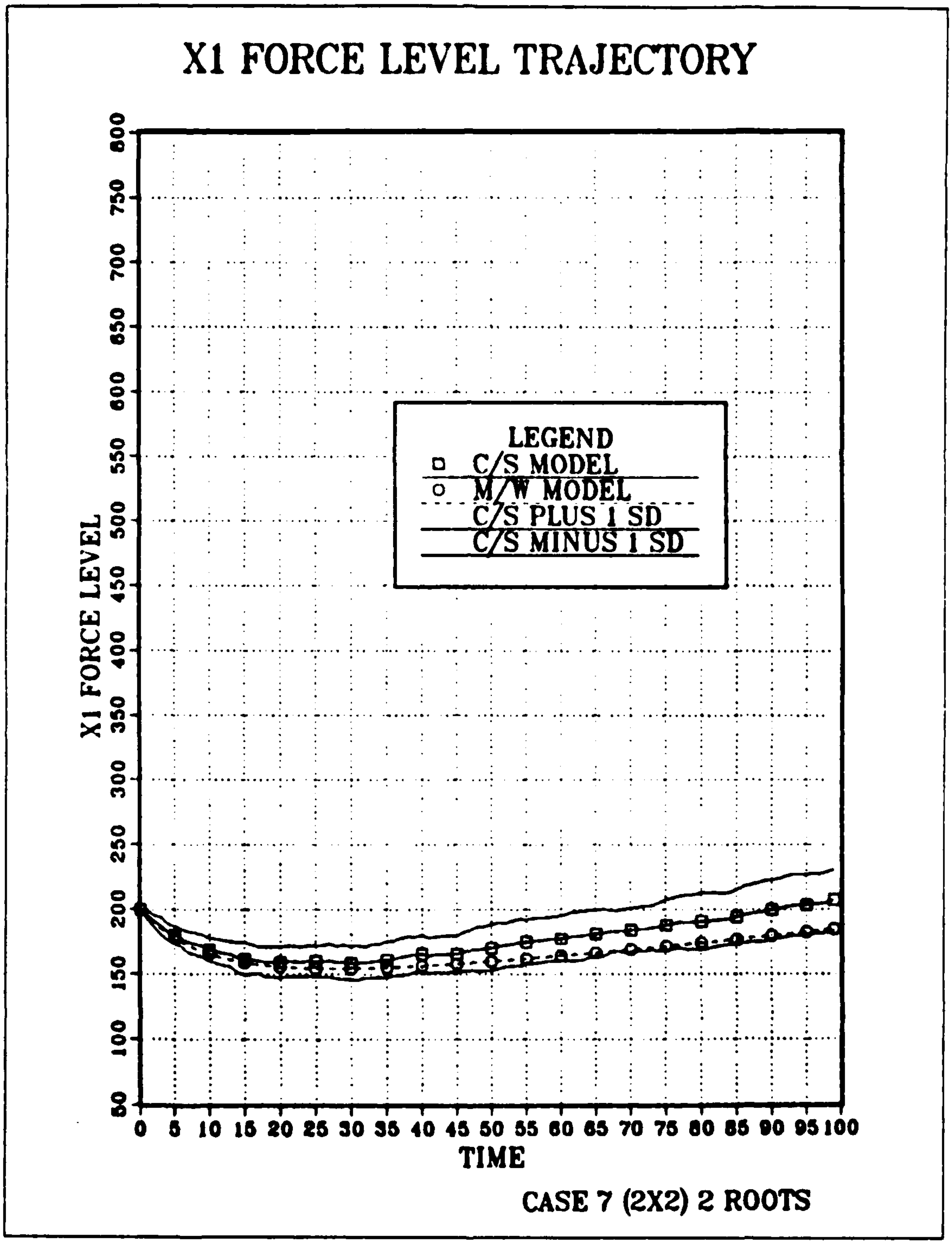

Figure E.29 X1 Force Level Trajectory Over Time For Case Seven.

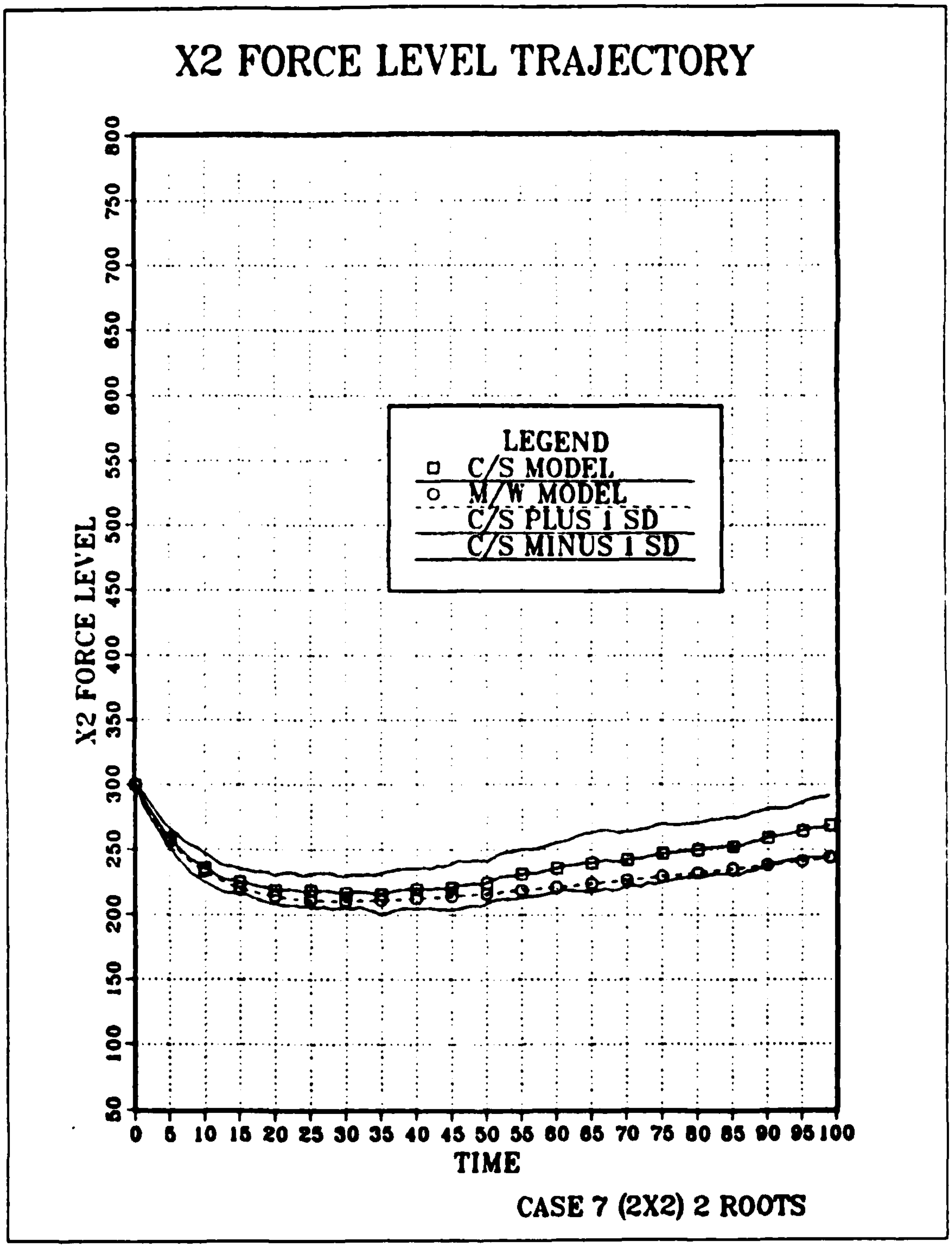

Figure E.30 X2 Force Level Trajectory Over Time For Case Seven.

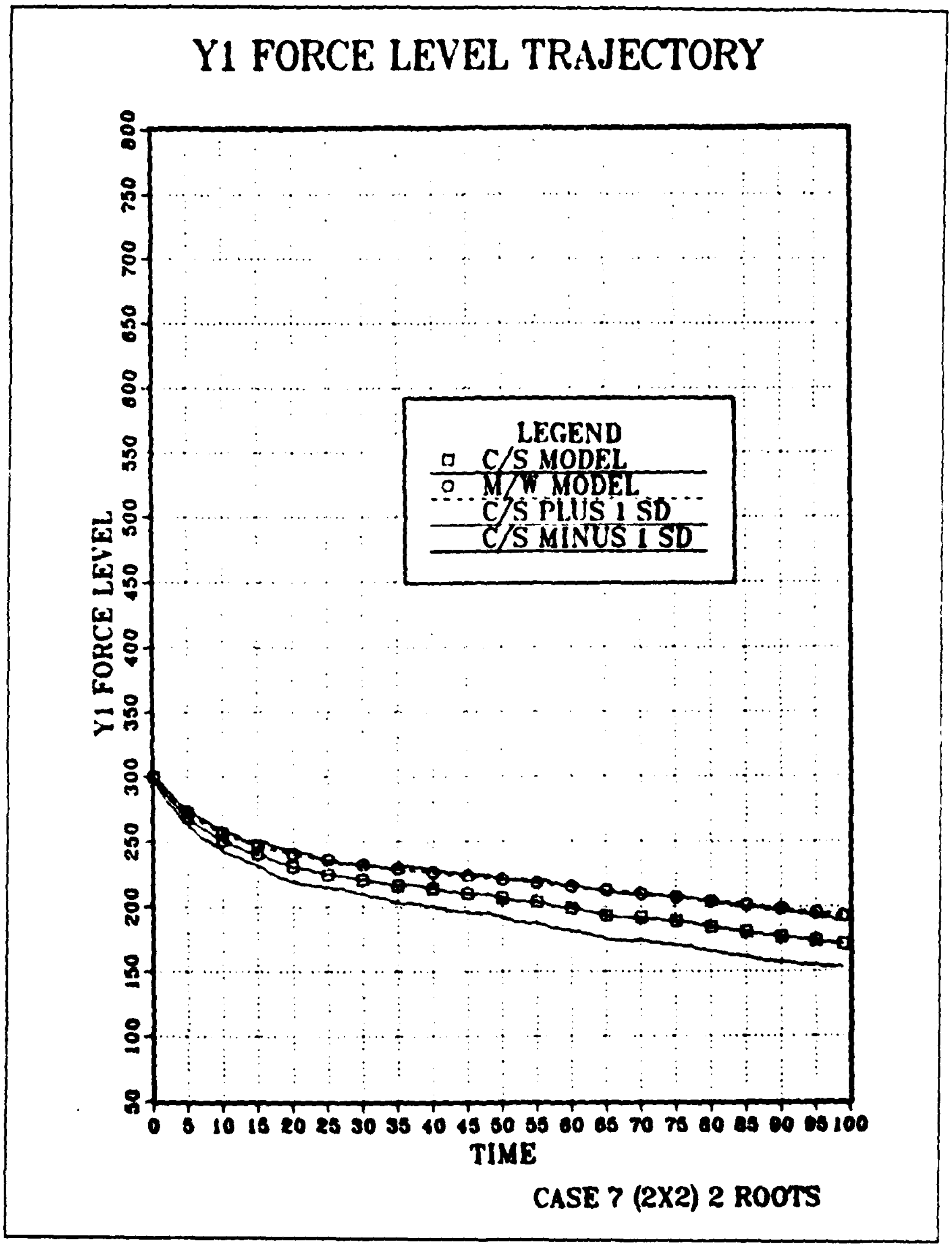

Figure E.31 Y1 Force Level Trajectory Over Time For Case Seven.

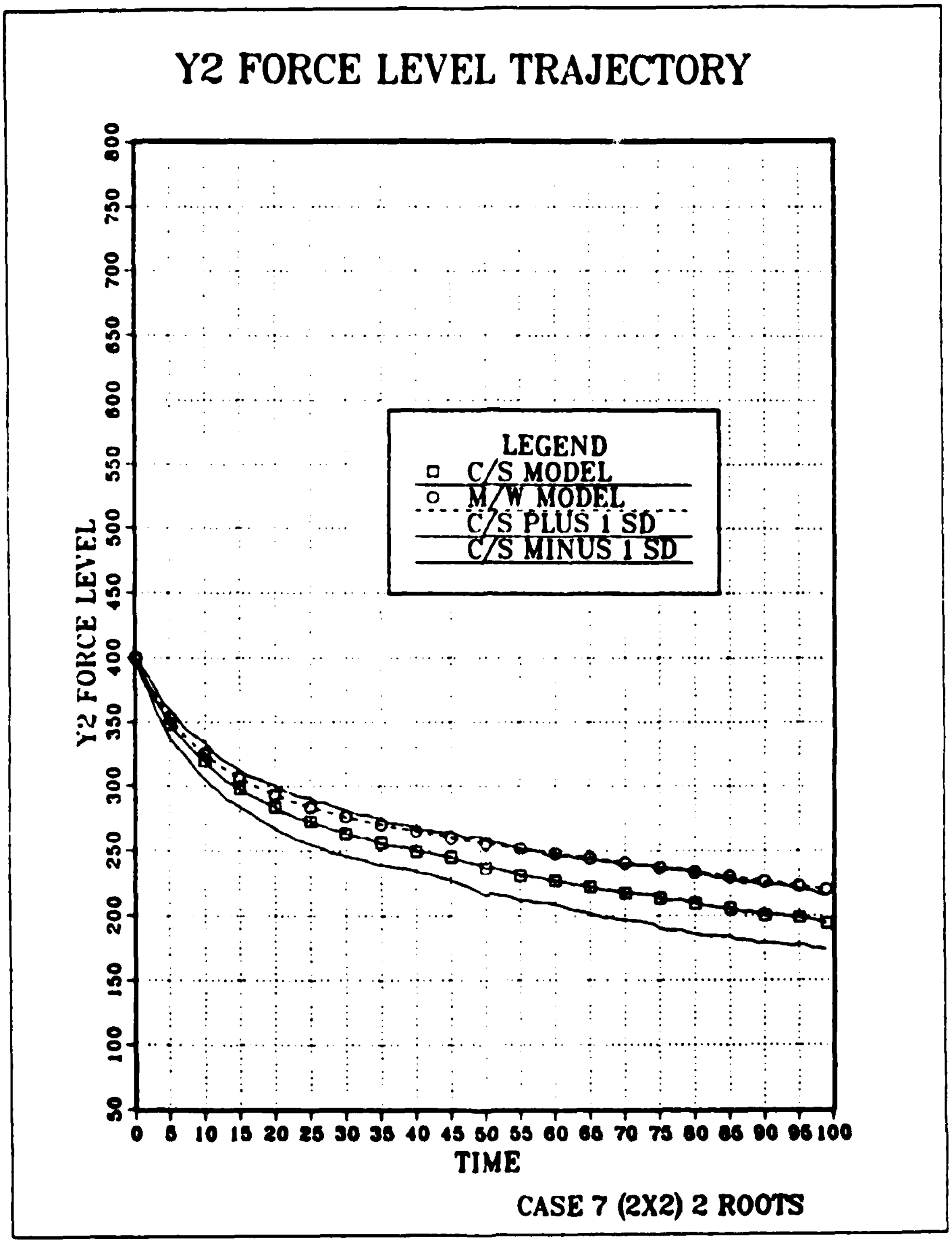

Figure E.32 Y2 Force Level Trajectory Over Time For Case Seven.

TABLE 17

INPUT DATA SET FOR C/S MODEL CASE 8

```
02/01/87  TEST OF (2X2), CASE 8; 1 STABLE ROOT
  99
    2  200  200    0  1.0  1.0  1.0  0.100  0.120  0.000   51   85    0
0.04000 0.06000 0.00000
0.05000 0.07000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00030 0.00000
0.00010 0.00040 0.00000
0.00000 0.00000 0.00000
    2  300  300    0  1.0  1.0  1.0  0.100  0.150  0.000   71   76    0
0.06000 0.04000 0.00000
0.05000 0.03000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00020 0.00000
0.00030 0.00020 0.00000
0.00000 0.00000 0.00000
  345215789.D0    45635761.D0    89342761.D0
```

TABLE 18

INPUT DATA SET FOR M/W MODEL CASE 8

```
02/01/87  TEST OF (2X2), CASE 8; 1 STABLE ROOT
  99
    2 200. 200.   0.  0.100  0.120  0.000   51   85    0
0.03003 0.04312 0.00000
0.03652 0.04845 0.00000
0.00000 0.00000 0.00000
0.00015 0.00021 0.00000
0.00007 0.00028 0.00000
0.00000 0.00000 0.00000
    2 300. 300.   0.  0.100  0.150  0.000   71   76    0
0.04653 0.03115 0.00000
0.03147 0.01856 0.00000
0.00000 0.00000 0.00000
0.00015 0.00016 0.00000
0.00019 0.00013 0.00000
0.00000 0.00000 0.00000
```

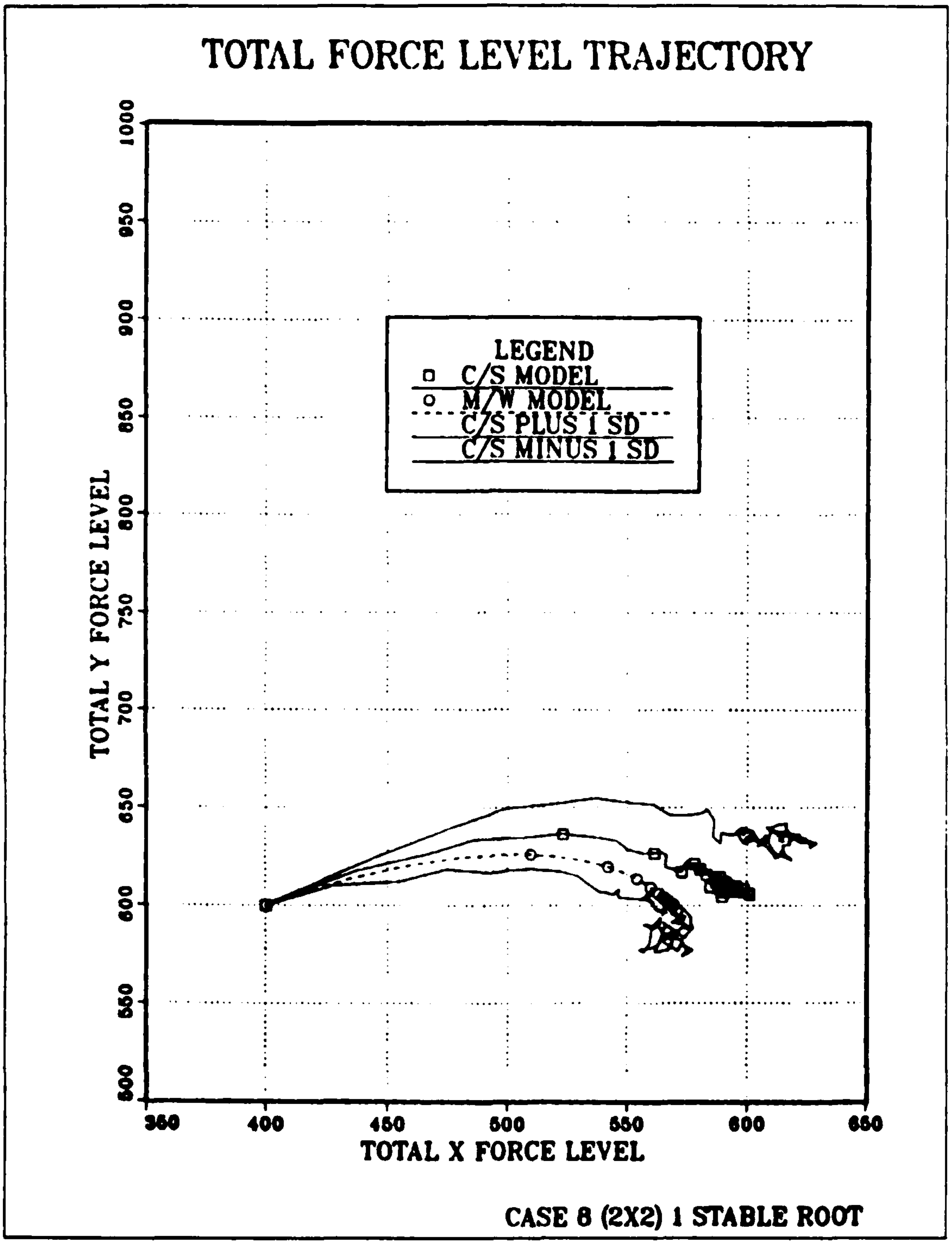

Figure E.33 Total Force Level Trajectory For Case Eight.

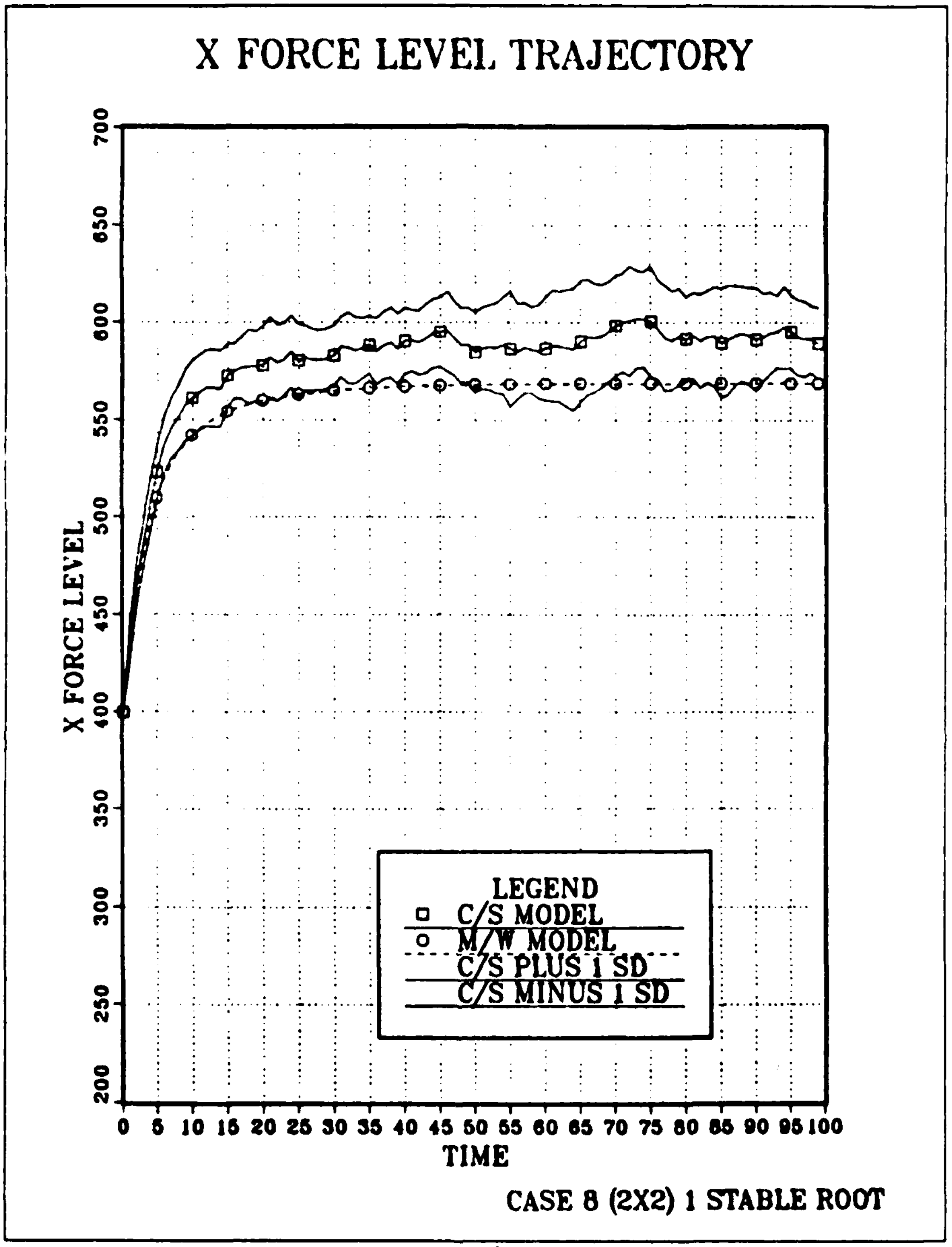

Figure E.34 X Force Level Trajectory Over Time For Case Eight.

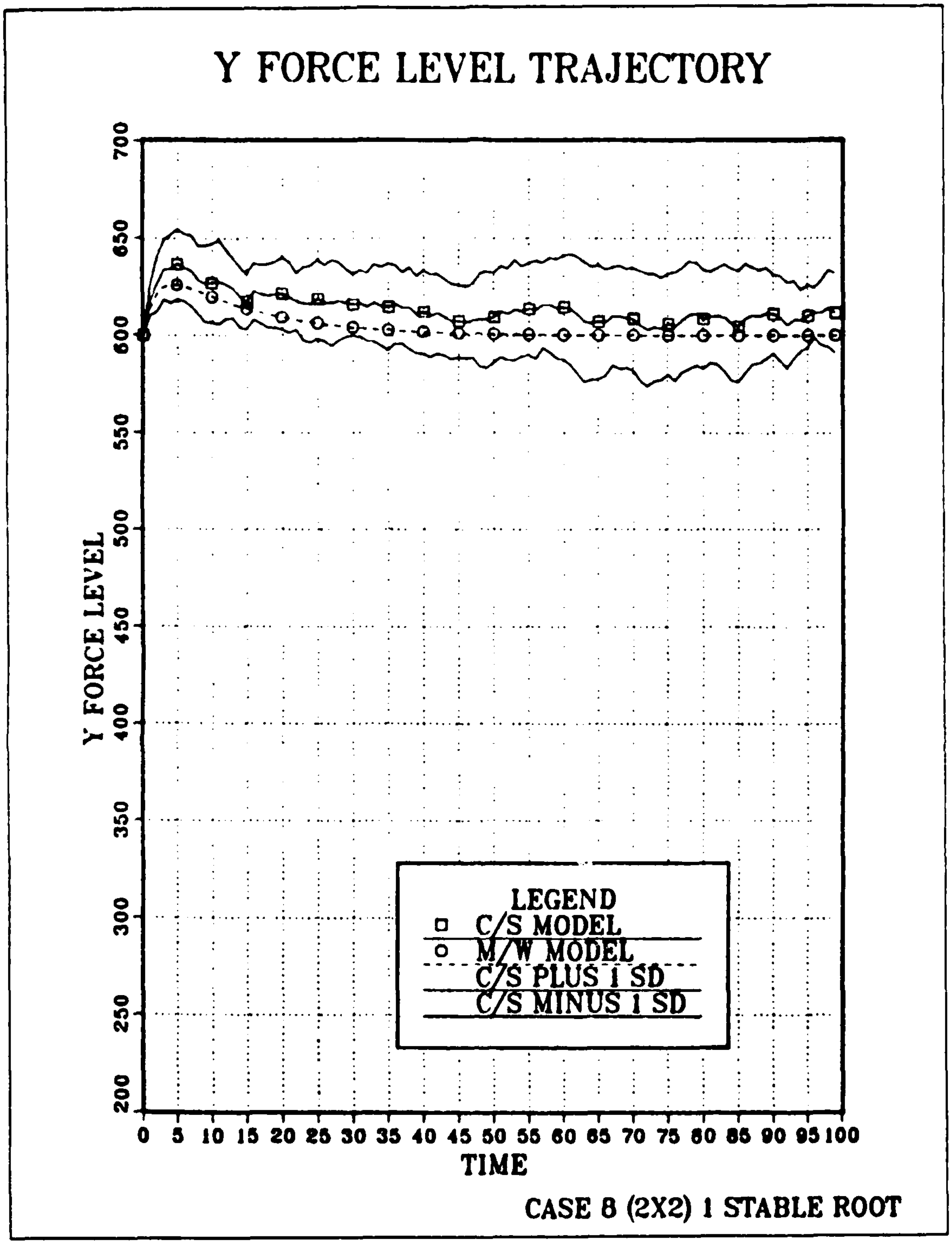

Figure E.35 Y Force Level Trajectory Over Time For Case Eight.

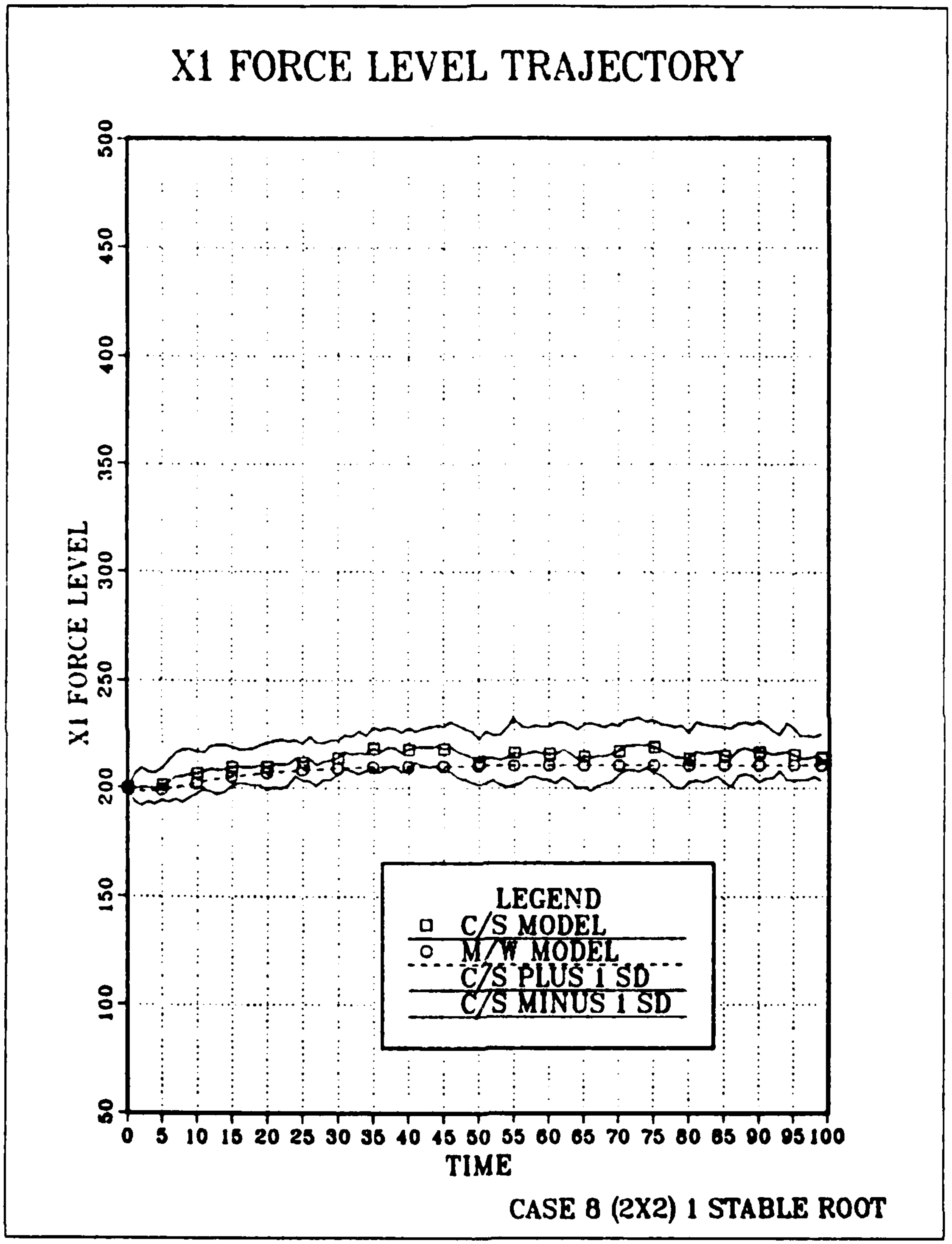

Figure E.36 X1 Force Level Trajectory Over Time For Case Eight.

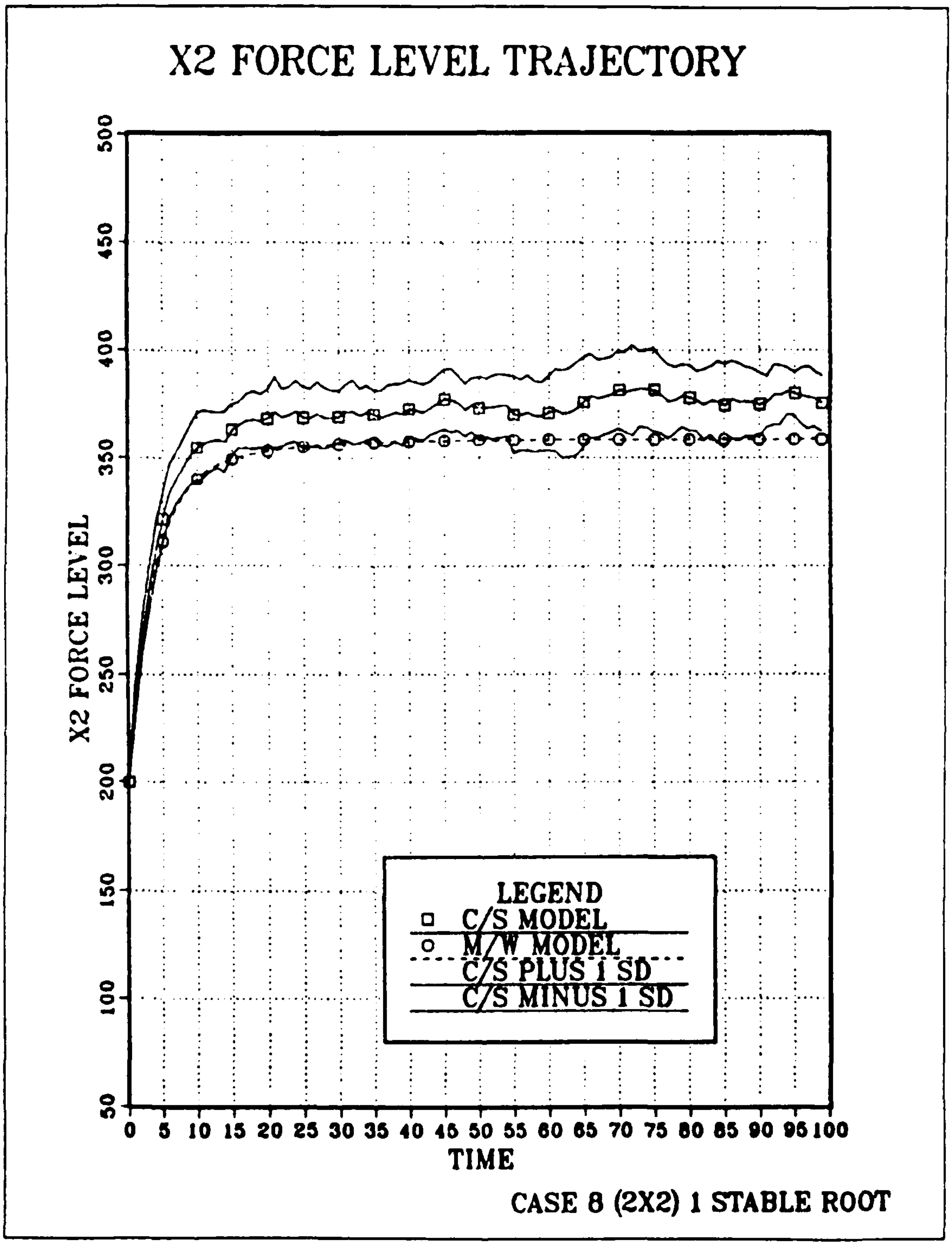

Figure E.37 X2 Force Level Trajectory Over Time For Case Eight.

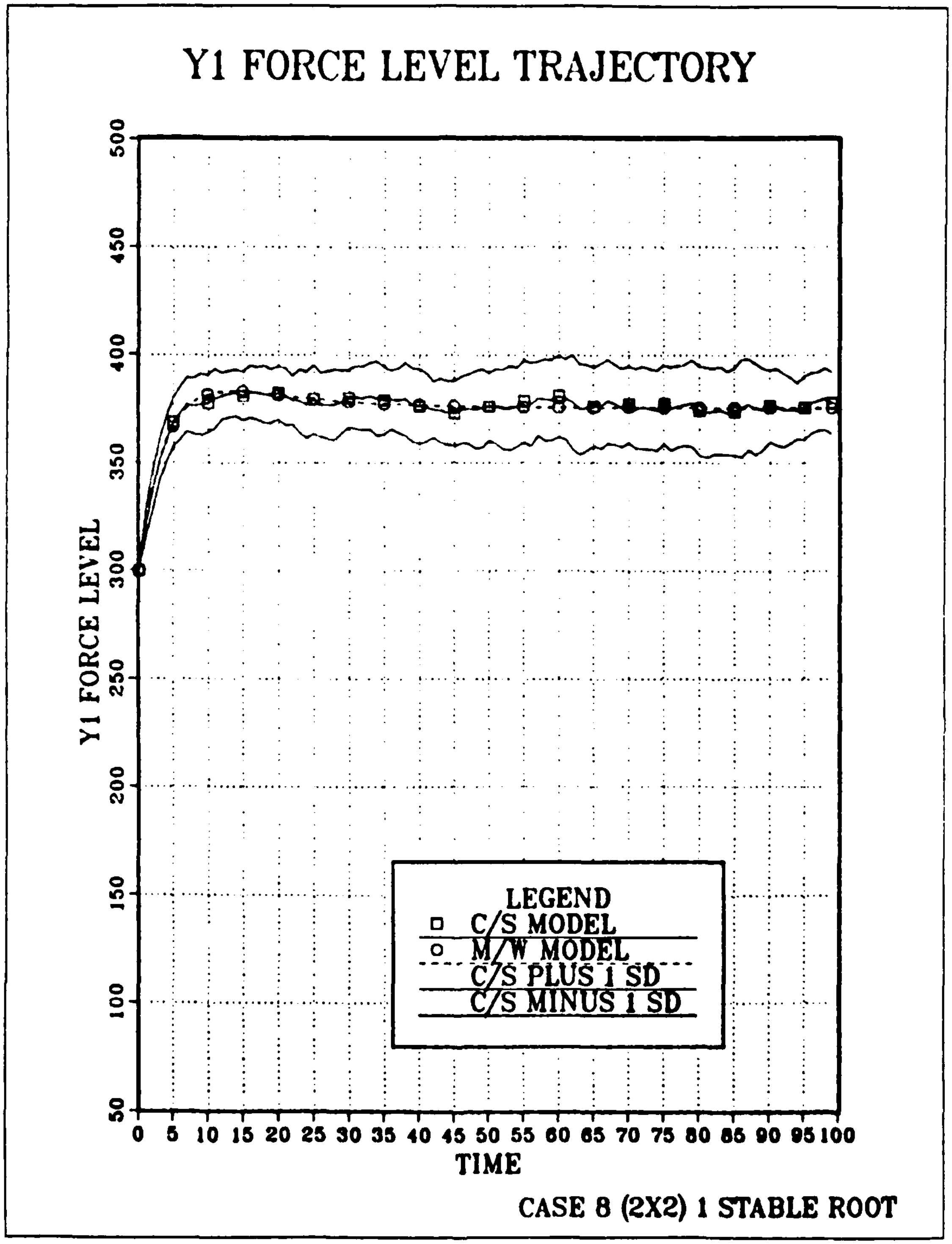

Figure E.38 Y1 Force Level Trajectory Over Time For Case Eight.

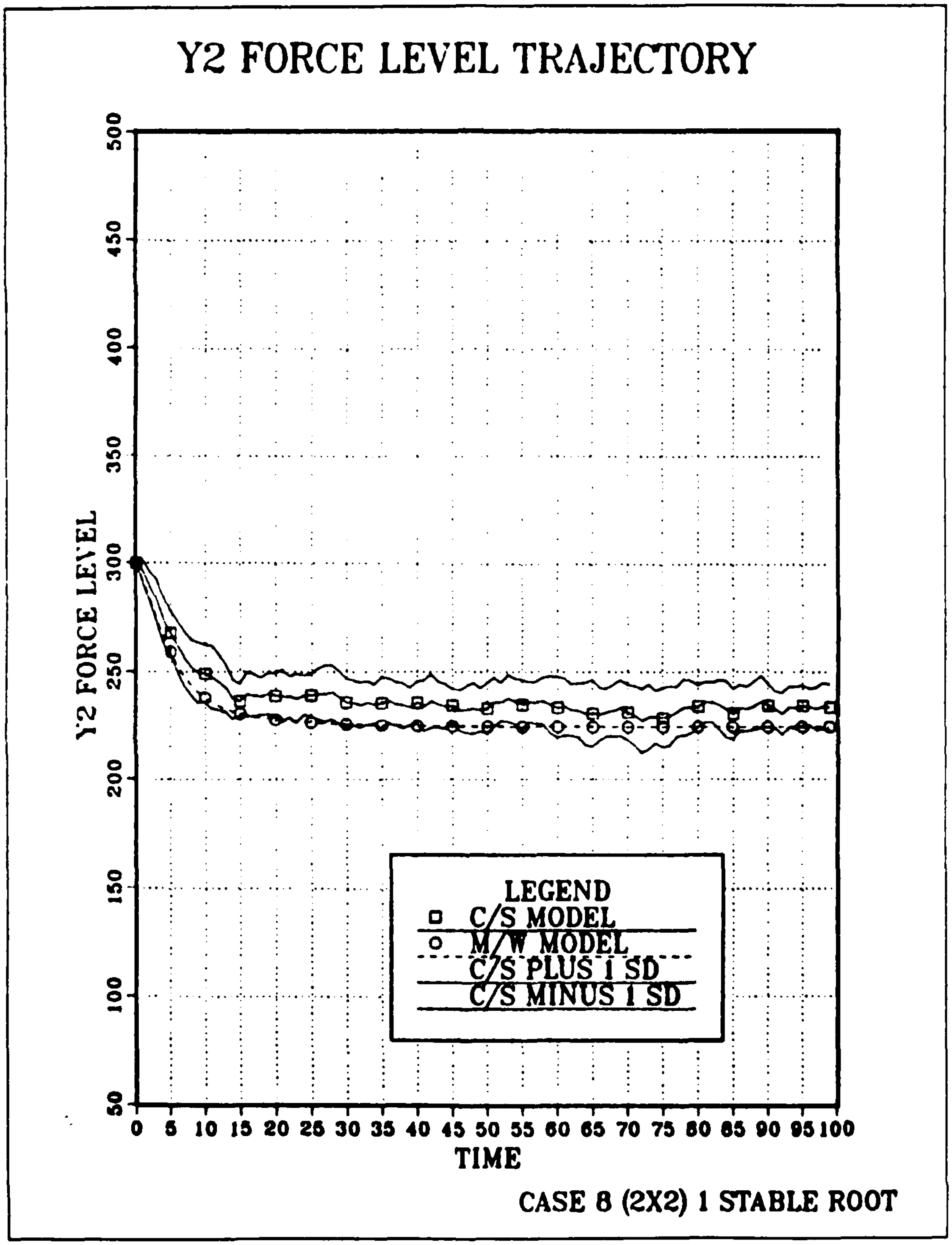

Figure E.39 Y2 Force Level Trajectory Over Time For Case Eight.

TABLE 19

INPUT DATA SET FOR C/S MODEL CASE 9

```
02/01/87  TEST OF (2X2), CASE 9, 1 UNSTABLE
  99
    2  400  400    0  1.0  1.0  1.0  0.050  0.030  0.000   55   60    0
0.08000 0.11000 0.00000
0.12000 0.07000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00030 0.00000
0.00020 0.00020 0.00000
0.00000 0.00000 0.00000
    2  400  400    0  1.0  1.0  1.0  0.070  0.020  0.000   65   54    0
0.10000 0.05000 0.00000
0.08000 0.15000 0.00000
0.00000 0.00000 0.00000
0.00020 0.00010 0.00000
0.00010 0.00030 0.00000
0.00000 0.00000 0.00000
  345215789.D0    45635761.D0    89342761.D0
```

TABLE 20

INPUT DATA SET FOR M/W MODEL CASE 9

```
02/01/87  TEST OF (2X2), CASE 9, 1 UNSTABLE ROOT
  99
    2 400. 400.   0.  0.050  0.030  0.000   55   60    0
0.04814 0.06804 0.00000
0.07278 0.04330 0.00000
0.00000 0.00000 0.00000
0.00012 0.00019 0.00000
0.00012 0.00012 0.00000
0.00000 0.00000 0.00000
    2 400. 400.   0.  0.070  0.020  0.000   65   54    0
0.06706 0.03417 0.00000
0.05404 0.10125 0.00000
0.00000 0.00000 0.00000
0.00013 0.00007 0.00000
0.00007 0.00020 0.00000
0.00000 0.00000 0.00000
```

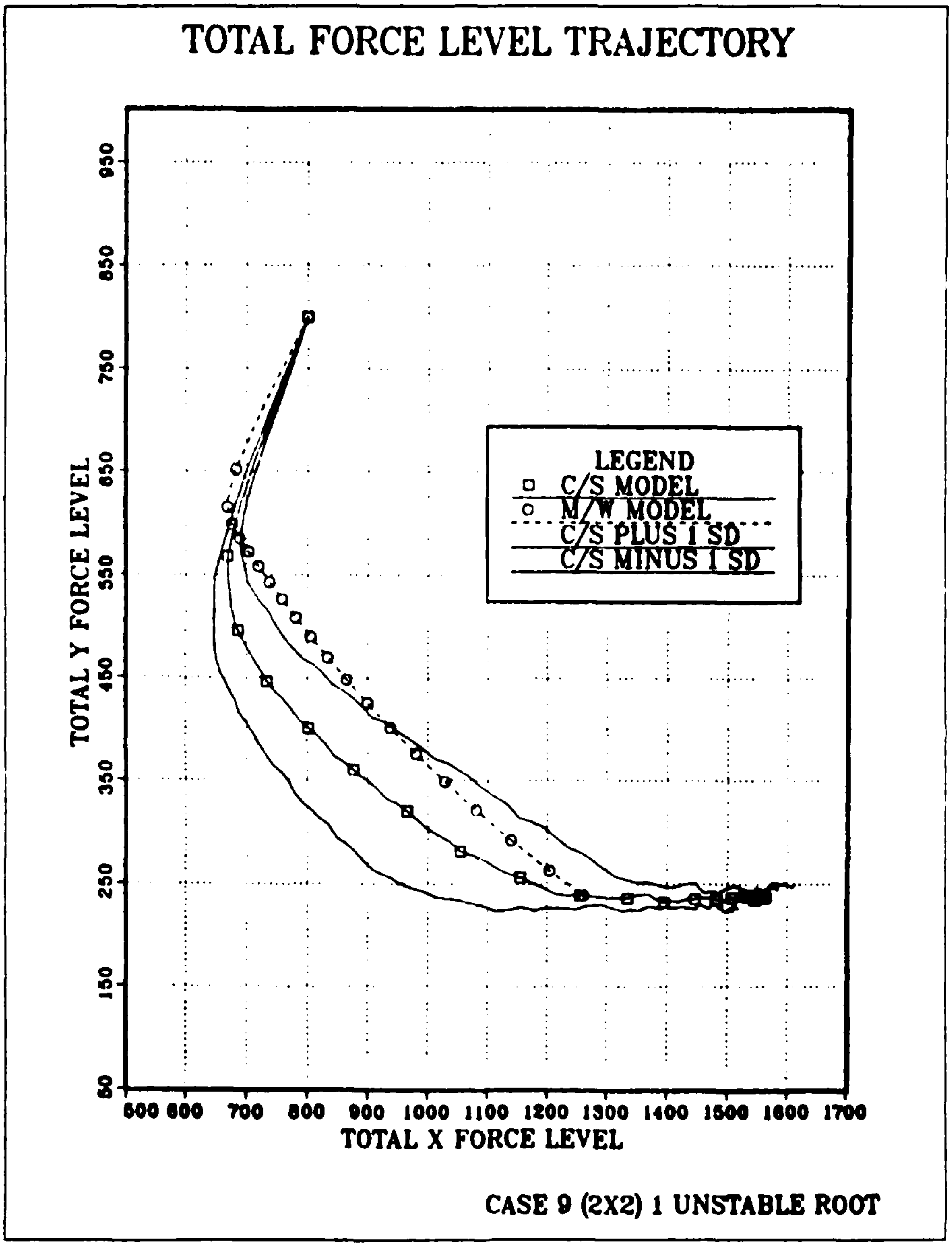

Figure E.40 Total Force Level Trajectory For Case Nine.

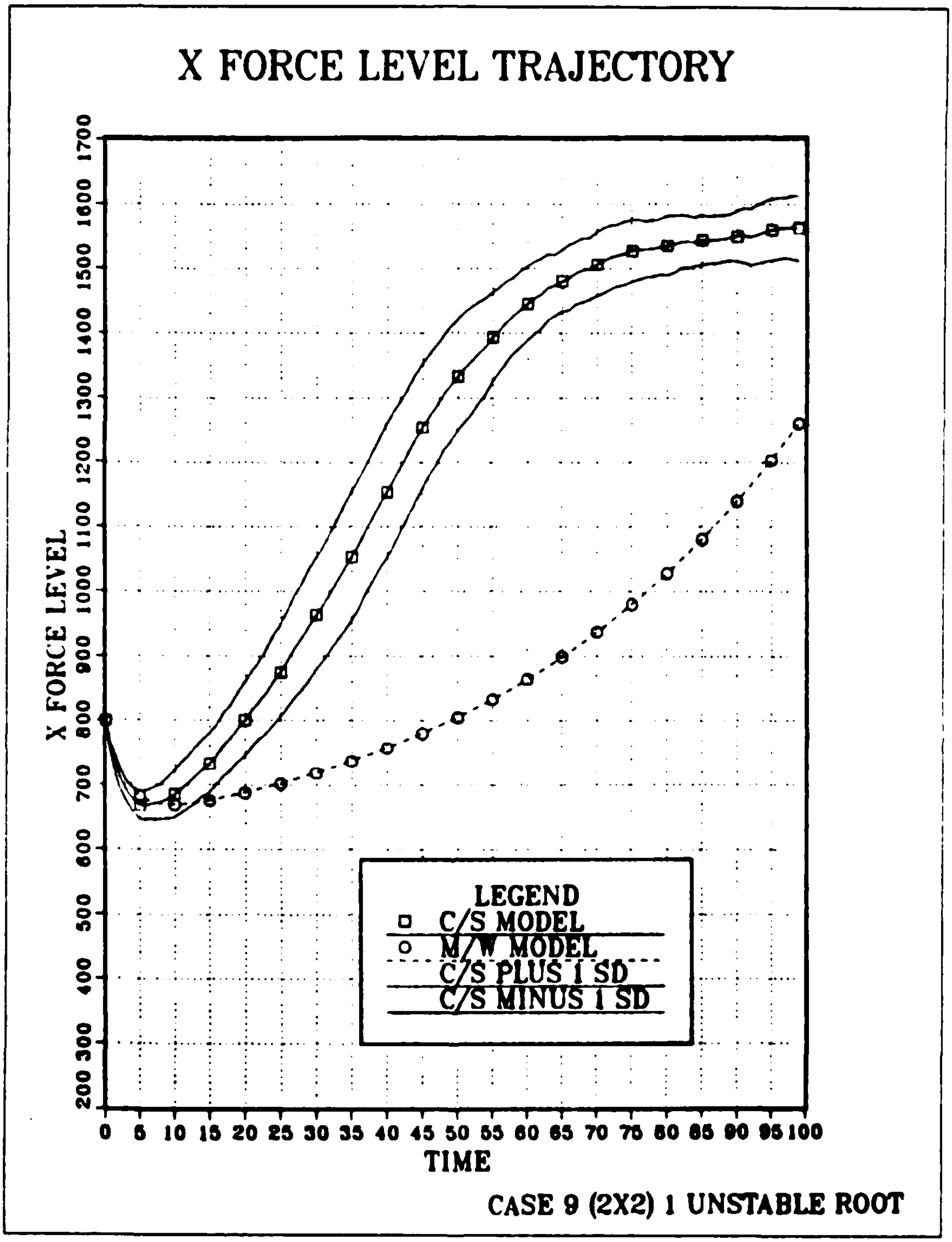

Figure E.41 X Force Level Trajectory Over Time For Case Nine.

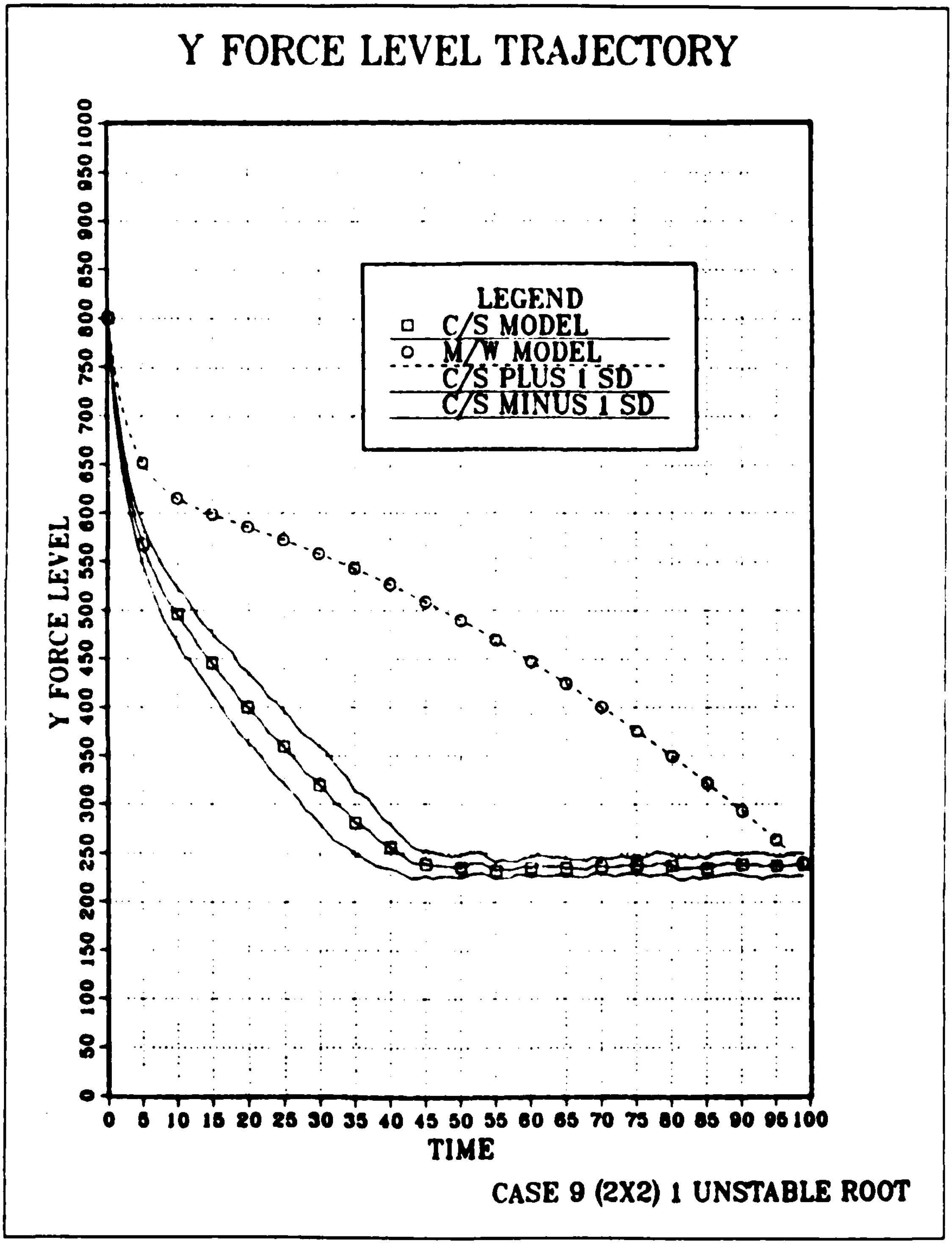

Figure E.42 Y Force Level Trajectory Over Time For Case Nine.

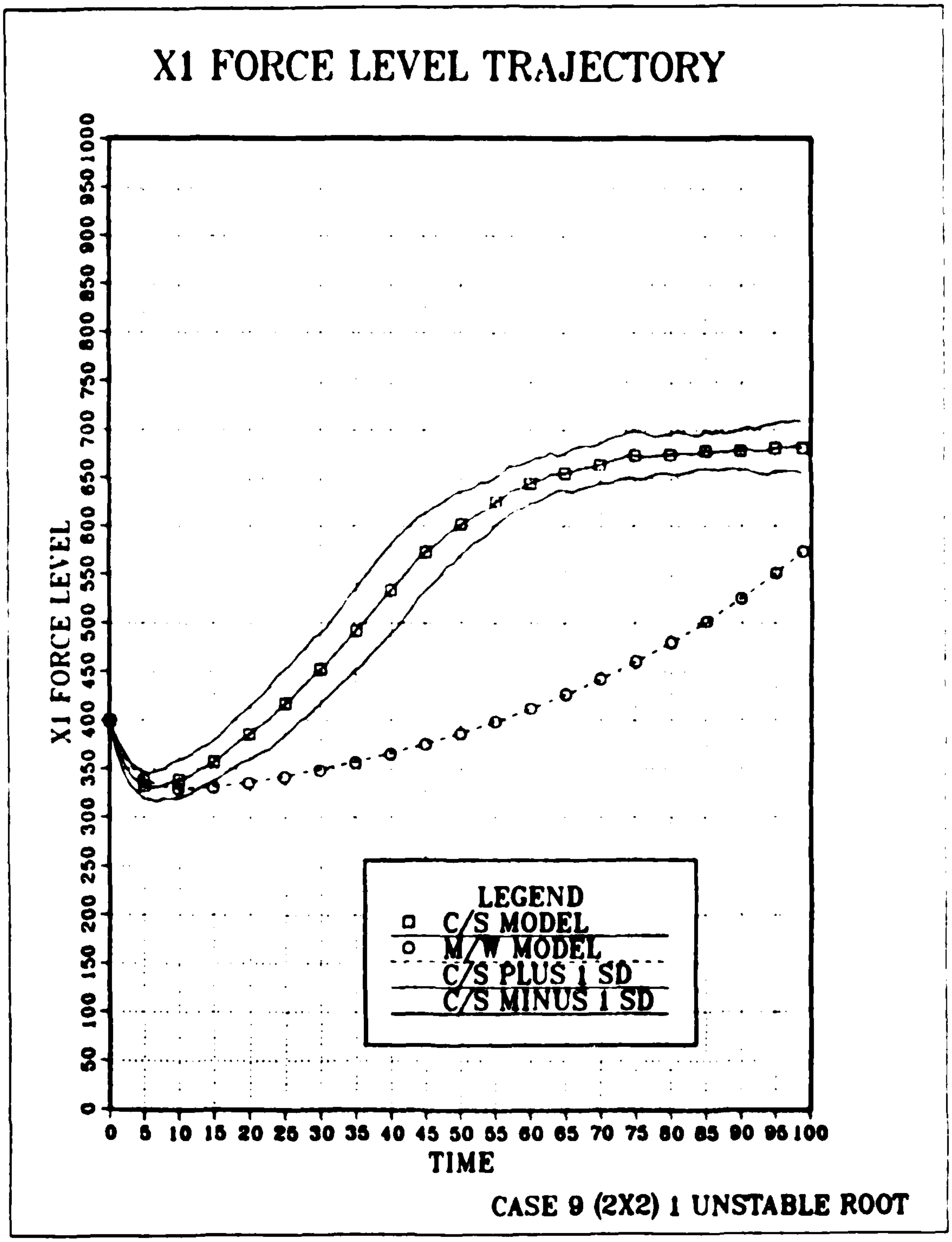

Figure E.43 X1 Force Level Trajectory Over Time For Case Nine.

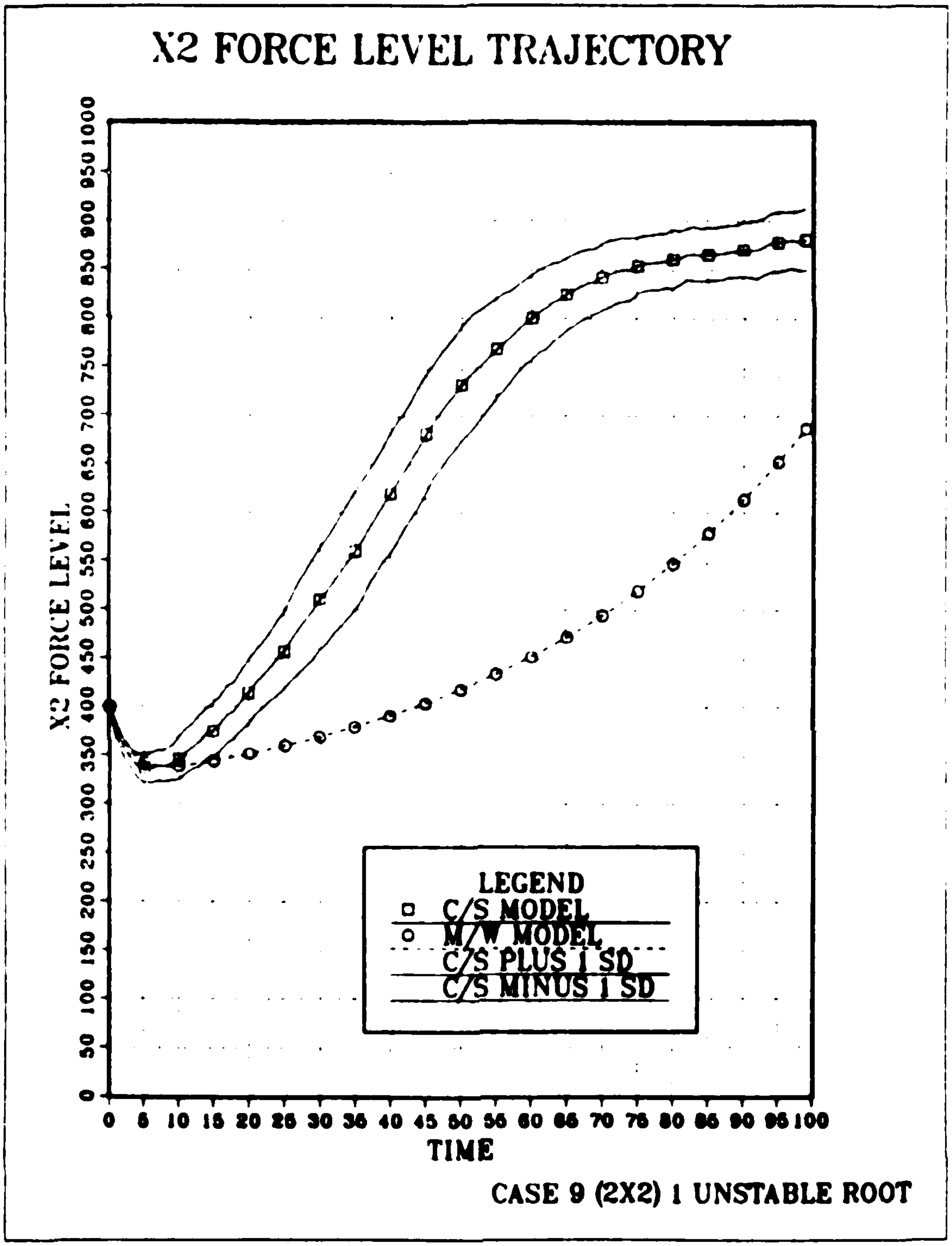

Figure E.44 X2 Force Level Trajectory Over Time For Case Nine.

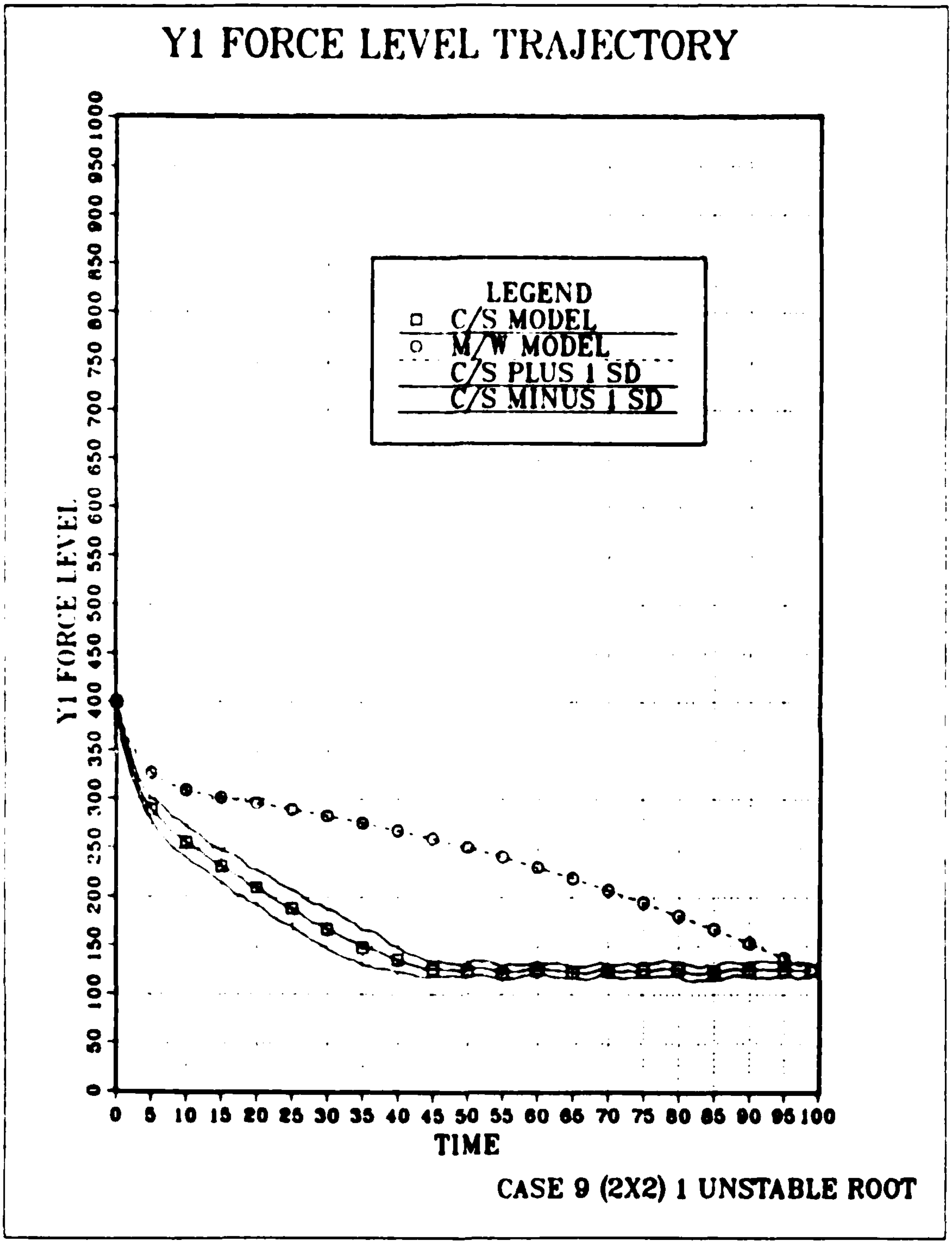

Figure E.45 Y1 Force Level Trajectory Over Time For Case Nine.

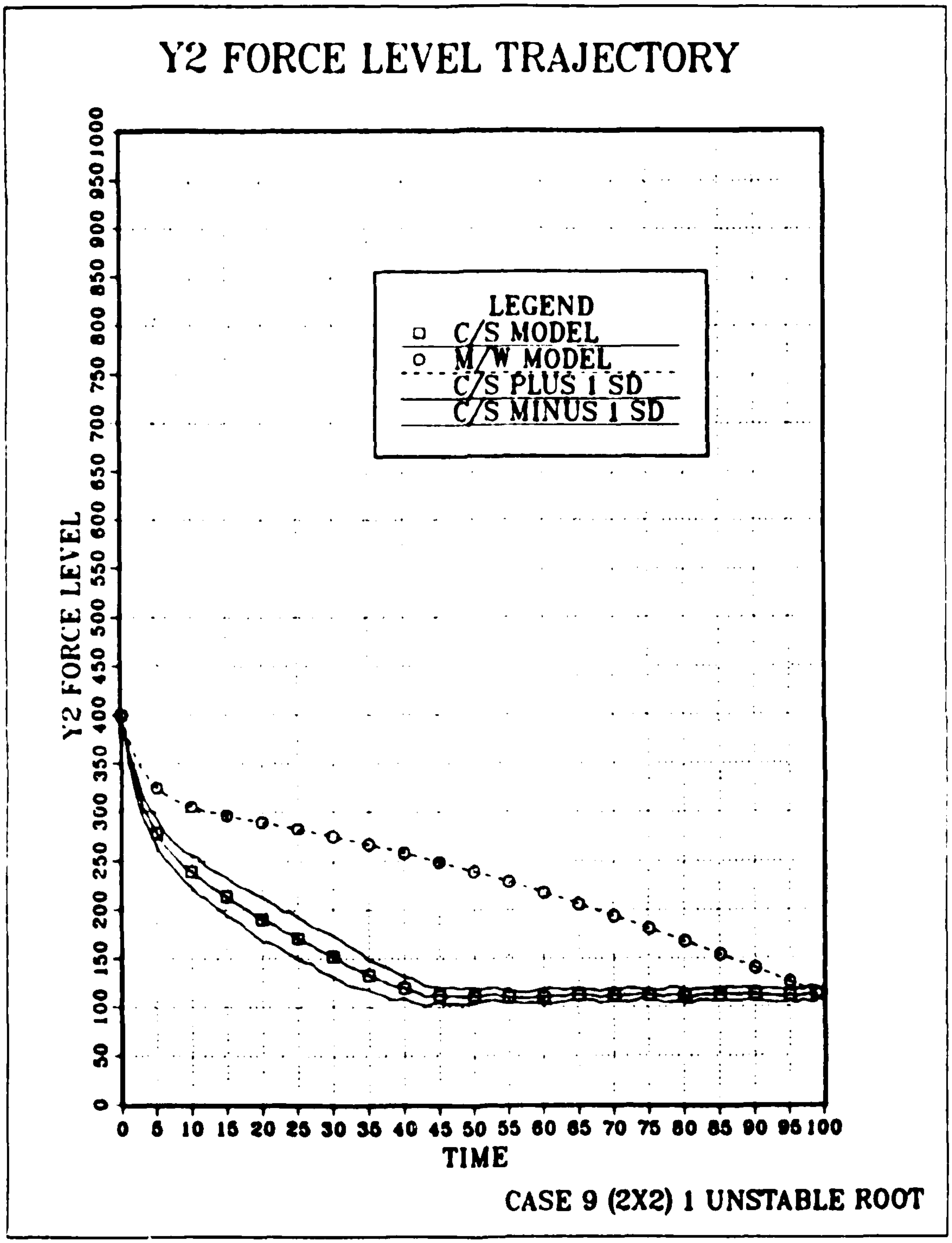

Figure E.46 Y2 Force Level Trajectory Over Time For Case Nine.

TABLE 21

INPUT DATA SET FOR C/S MODEL CASE 10

```
02/02/87  TEST OF (3X3)
    99
     3  200  150  175  1.0  1.0  1.0  0.005  0.010  0.020   4   4   11
0.06000 0.02000 0.02000
0.00000 0.00000 0.00000
0.03000 0.01000 0.02000
0.00000 0.00000 0.00000
0.00000 0.00030 0.00020
0.00000 0.00000 0.00010
     3  150  125  225  1.0  1.0  1.0  0.003  0.020  0.030   4   5   14
0.05000 0.01000 0.05000
0.00000 0.00000 0.00000
0.02000 0.01000 0.03000
0.00000 0.00000 0.00000
0.00000 0.00020 0.00030
0.00000 0.00000 0.00010
    345215789.D0    45635761.D0    89342761.D0
```

TABLE 22

INPUT DATA SET FOR M/W MODEL CASE 10

```
01/26/87  TEST OF (3X3)
    99
     3 200. 150. 175.  0.005  0.010  0.020   4   4   11
0.05836 0.01956 0.01916
0.00000 0.00000 0.00000
0.02701 0.00863 0.01745
0.00000 0.00000 0.00000
0.00000 0.00028 0.00019
0.00000 0.00000 0.00009
     3 150. 125. 225.  0.003  0.020  0.030   4   5   14
0.04884 0.00993 0.04893
0.00000 0.00000 0.00000
0.01884 0.00926 0.02661
0.00000 0.00000 0.00000
0.00000 0.00019 0.00028
0.00000 0.00000 0.00009
```

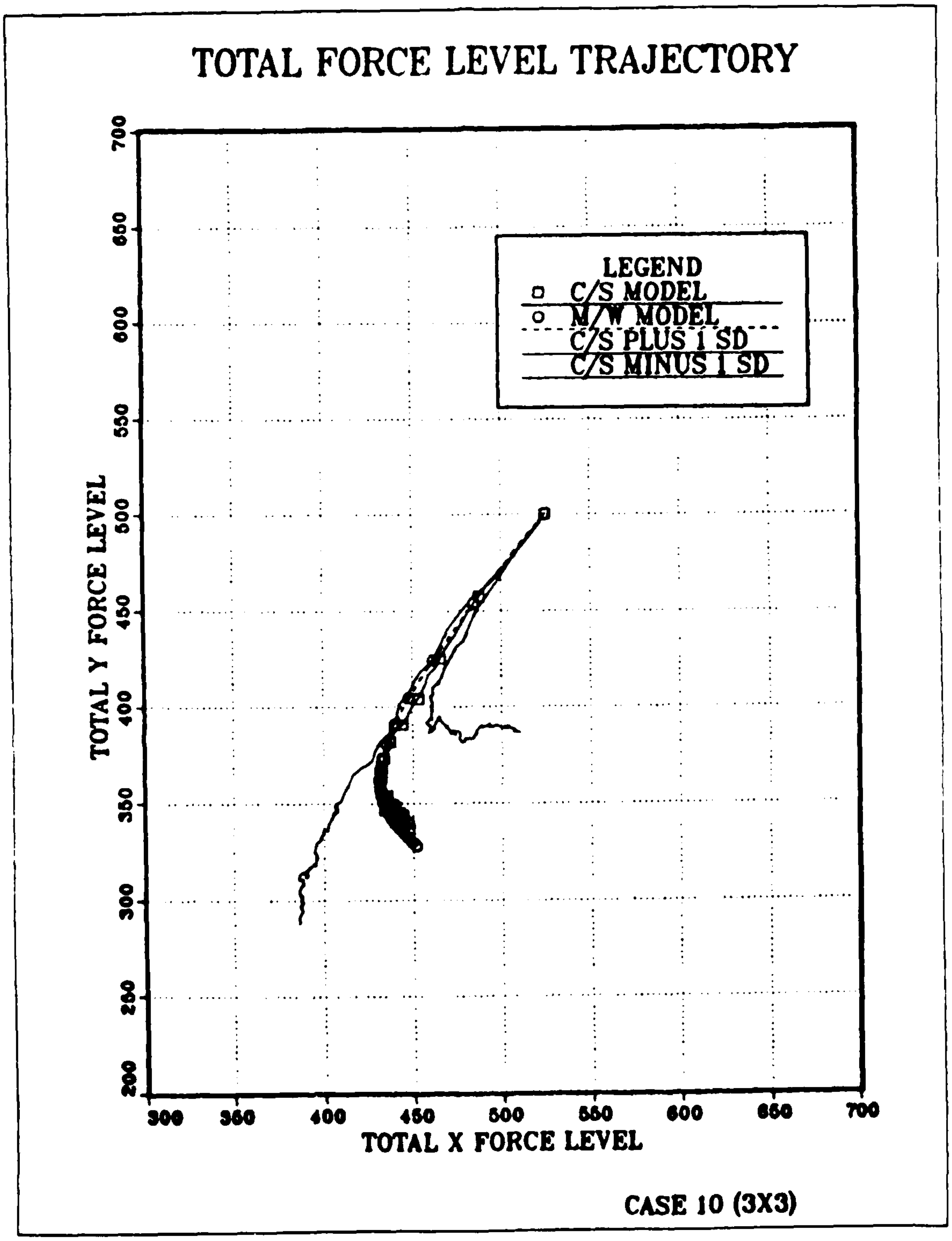

Figure E.47 Total Force Level Trajectory For Case Ten.

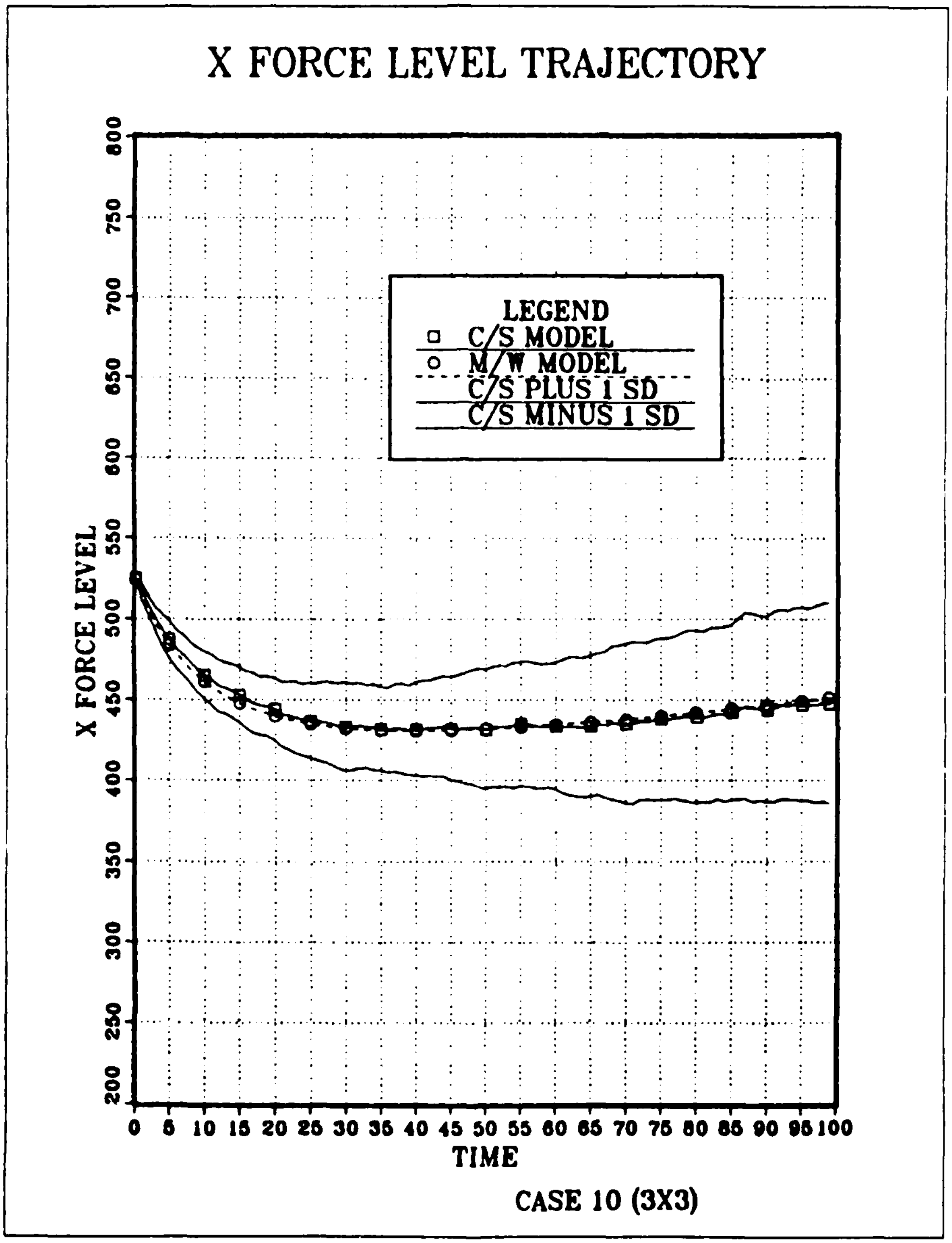

Figure E.48 X Force Level Trajectory Over Time For Case Ten.

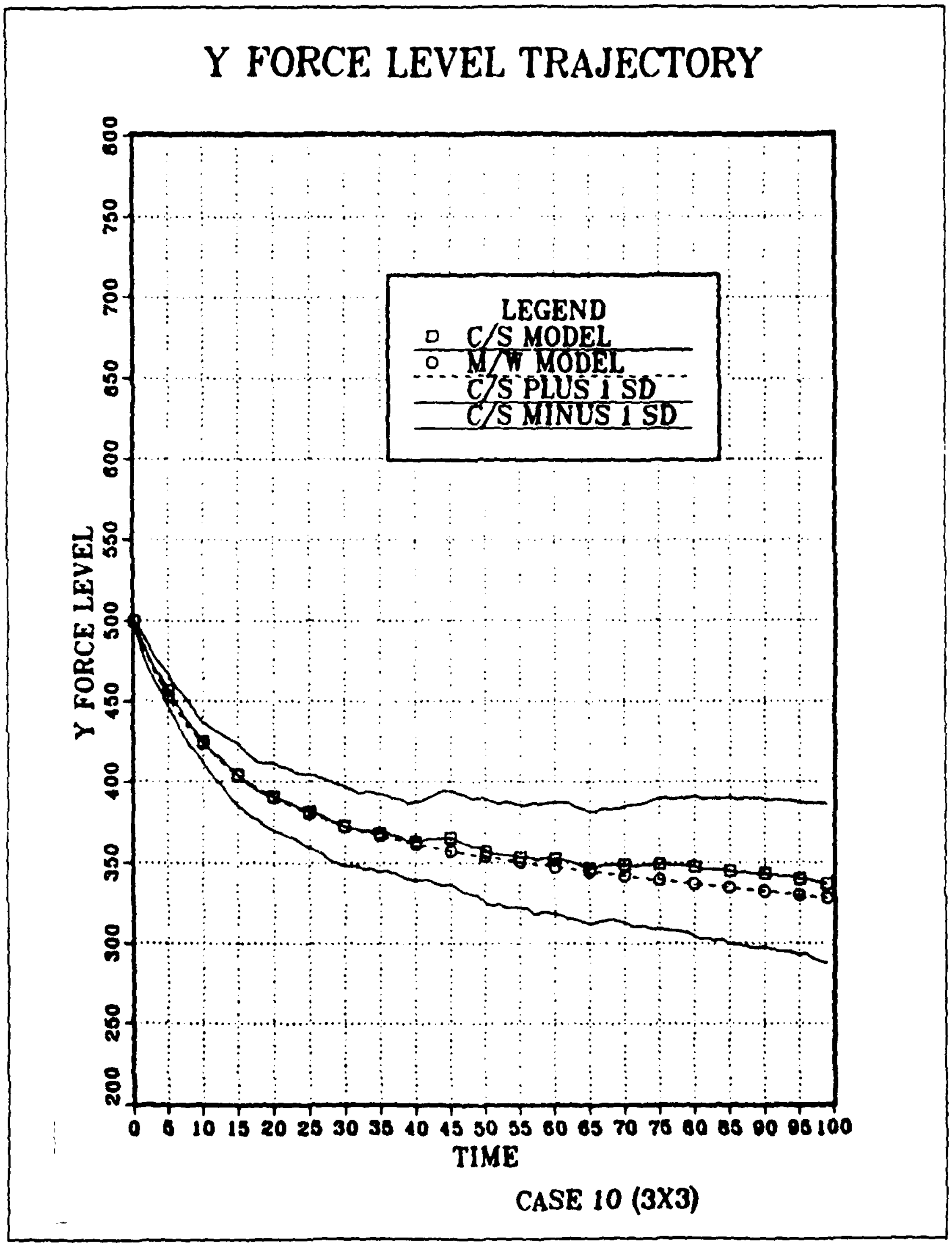

Figure E.49 Y Force Level Trajectory Over Time For Case Ten.

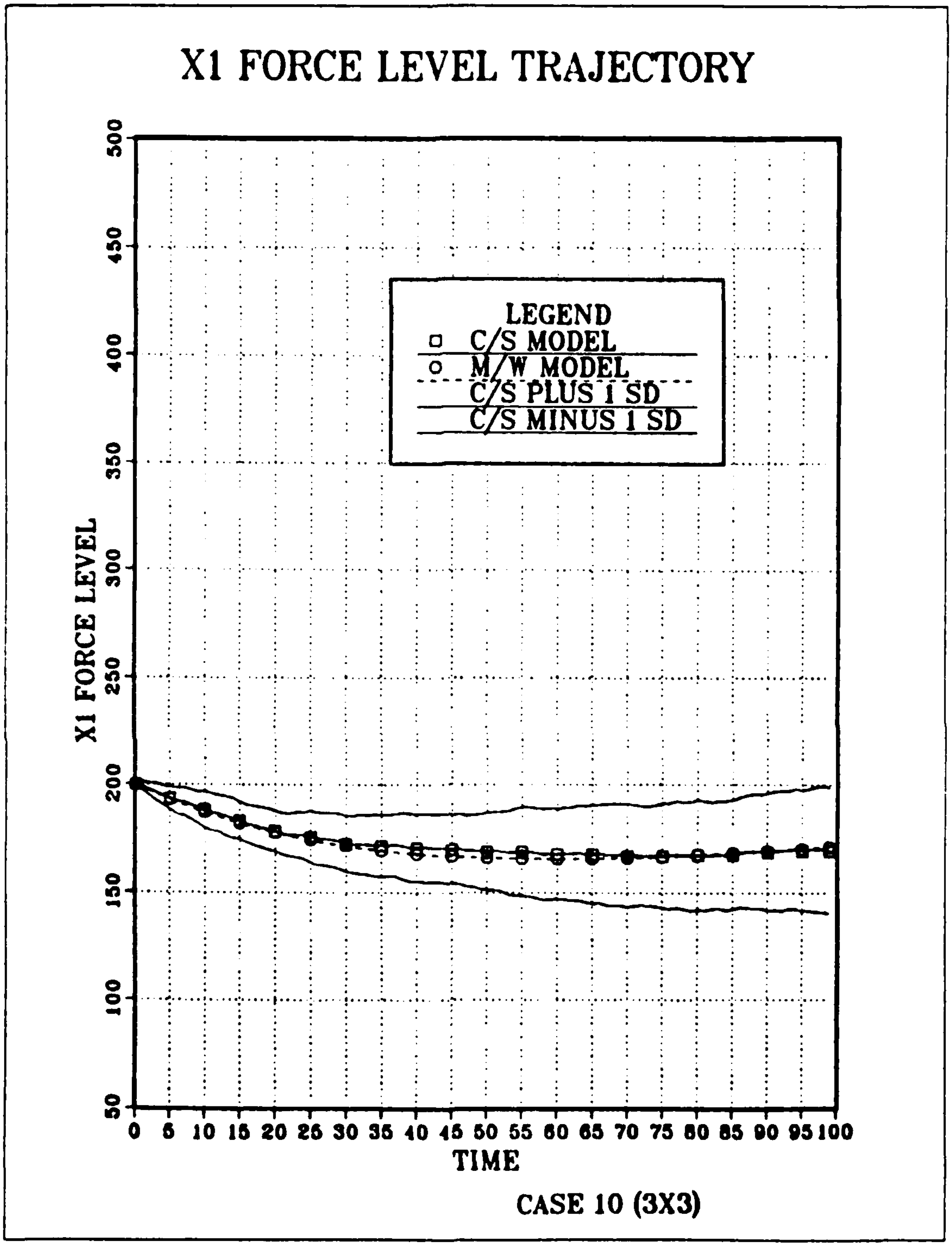

Figure E.50 X1 Force Level Trajectory Over Time For Case Ten.

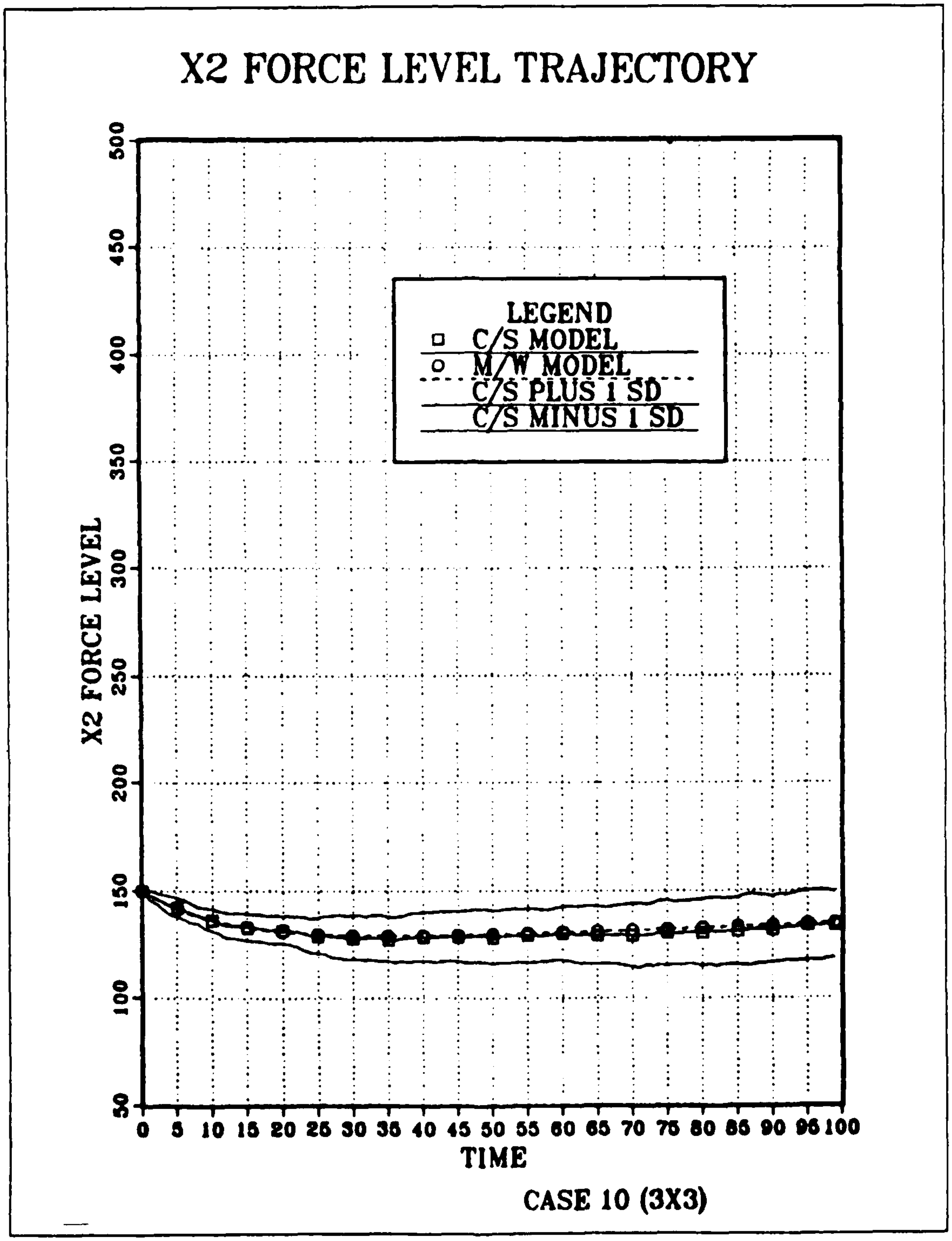

Figure E.51 X2 Force Level Trajectory Over Time For Case Ten.

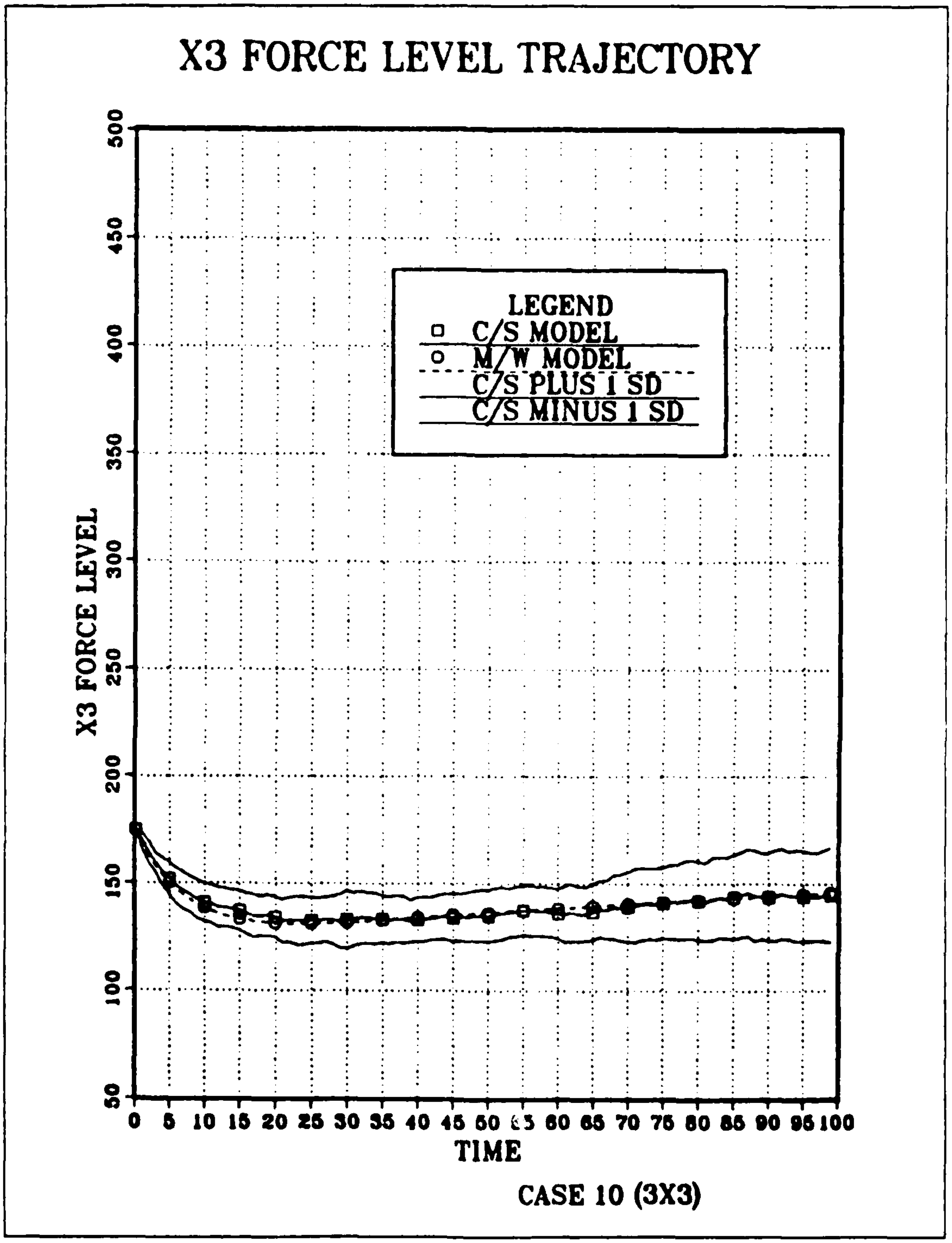

Figure E.52 X3 Force Level Trajectory Over Time For Case Ten.

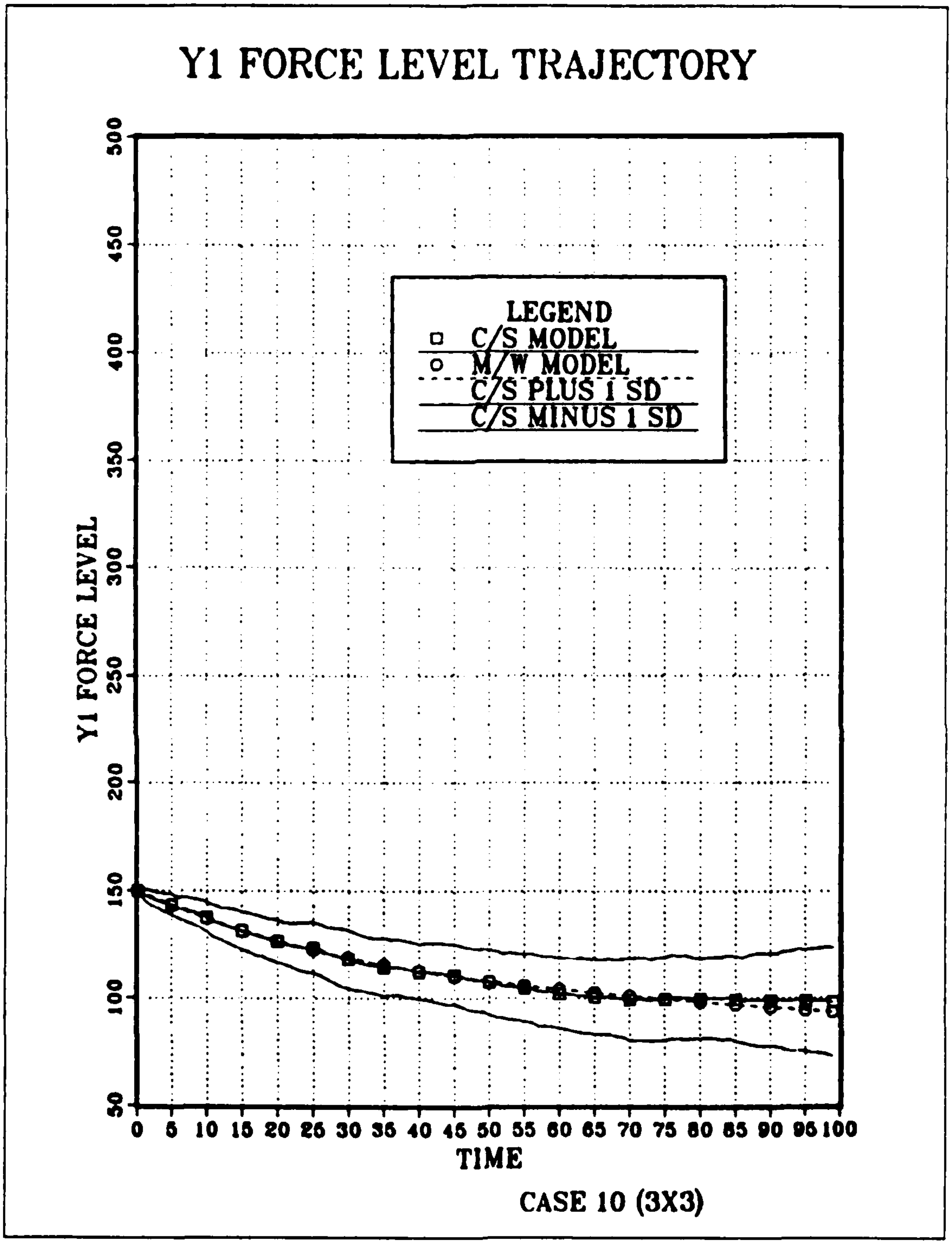

Figure E.53 Y1 Force Level Trajectory Over Time For Case Ten.

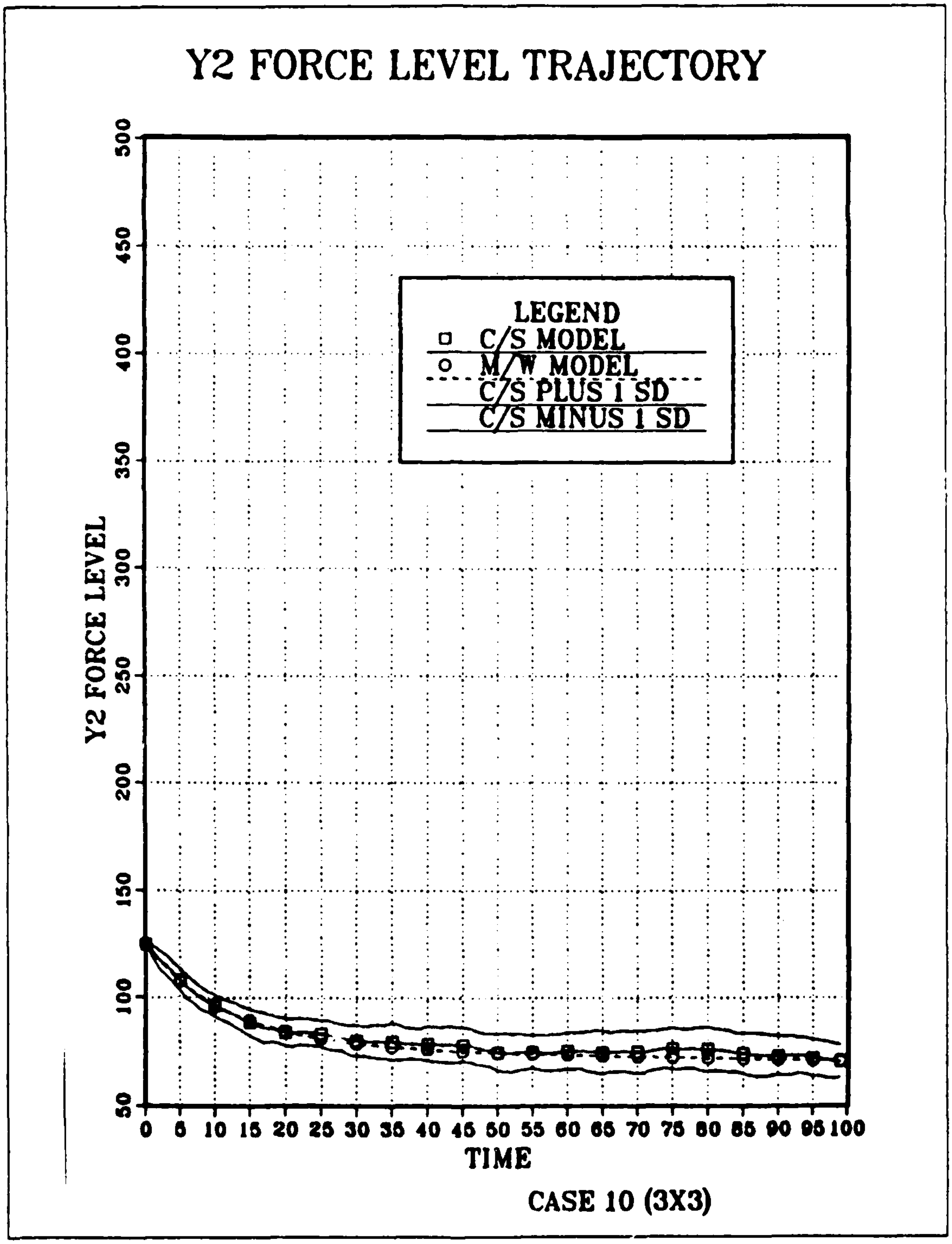

Figure E.54 Y2 Force Level Trajectory Over Time For Case Ten.

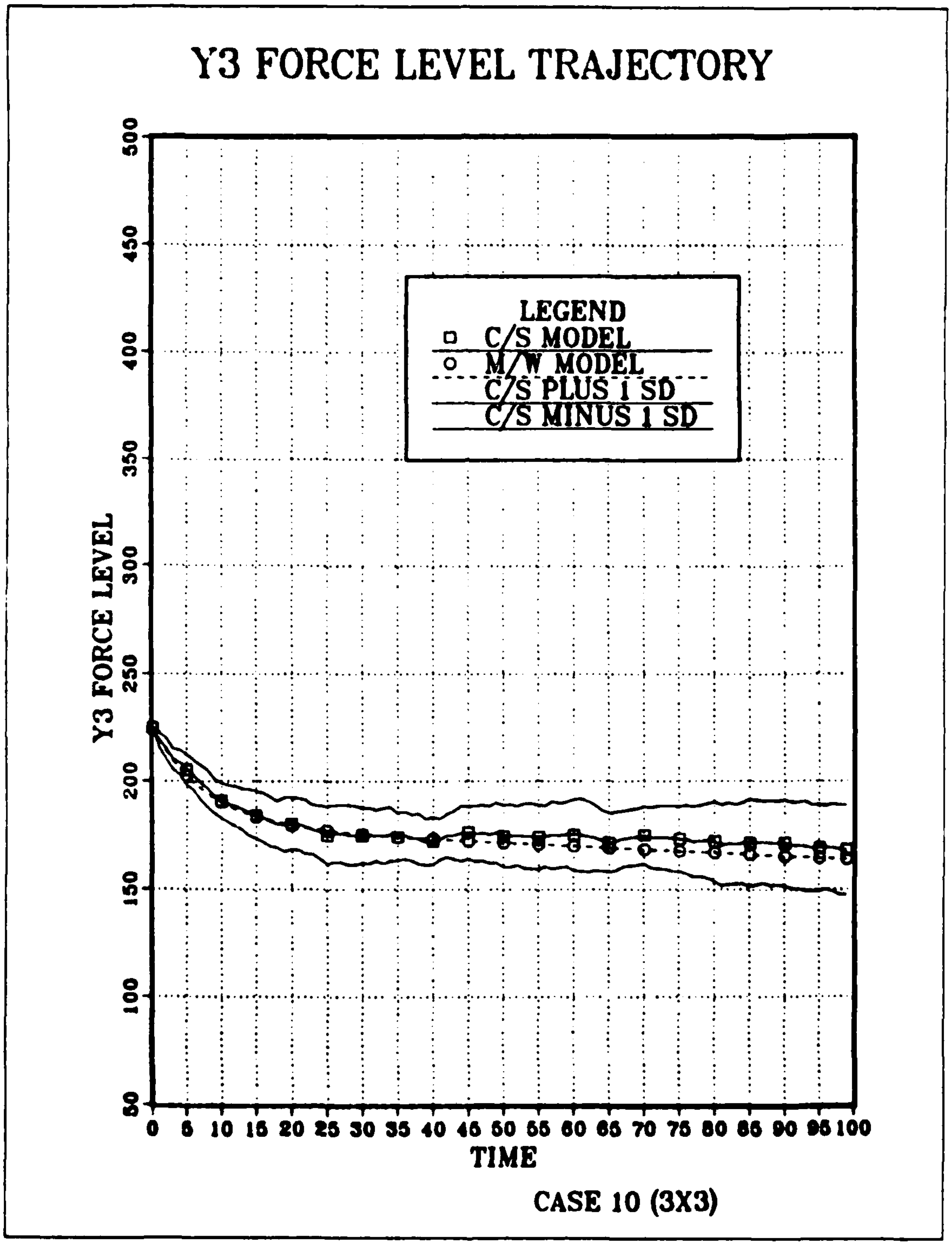

Figure E.55 Y3 Force Level Trajectory Over Time For Case Ten.

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